Illinois Institute of Technology

RADIATION BIOPHYSICS:
Electromagnetic Radiation

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Electromagnetic Radiation

- Much of this course deals with the interaction between electromagnetic radiation (usually ionizing) and matter
- So we need to review the properties of electromagnetic radiation
- EM radiation encompasses a wide range of energy / wavelength / frequency
  - Ionizing radiation: E = 3KeV - up, λ = 0.4 nm - down
  - Visible light is lower-energy (E~1eV, λ~500 nm)

The Electromagnetic Spectrum

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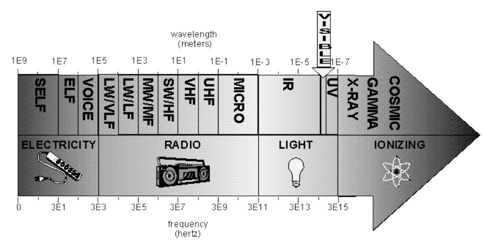
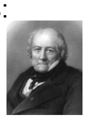
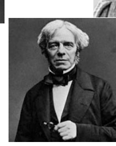
… or in graphical form



Laws of Electromagnetism

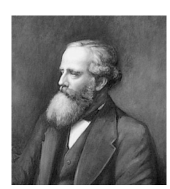
- The four major rules of electrodynamics:
  - Coulomb's law ($\boldsymbol{F} = kq_1q_2r^{-2}$)
  - Biot-Savart law
  - Faraday's law
  - Conservation of charge



Maxwell's contribution

- These rules predated Maxwell. He showed they could be made self-consistent by recognizing that a changing electric field induces a magnetic field; thus the integral and differential forms of Maxwell's equations.



What Maxwell's laws mean for radiation

- The electromagnetic field travels away from its source with velocity = $3 * 10^8$ m /sec. This turns out to be the velocity of light, so evidently light *is* an electromagnetic wave!
- Relationship between frequency and wavelength:
  $c = \nu\lambda$

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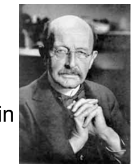
Planck's contribution



- Planck sought to understand radiation in a cavity by assuming that the atoms in the cavity were electromagnetic oscillators with characteristic frequencies and the oscillators would absorb and emit radiation
- He also posited that the oscillators were constrained to have energies
  $E = (n+1/2)h\nu$
  where ν = frequency, h = a constant

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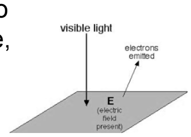
Einstein's photoelectric effect



- Energy of photons goes into enabling the electrons to escape from the surface, plus their kinetic energy after they do so:
  $E_{photon} = h\nu = E_0 + K_{max}$
- Here E_0 is the work function,
  K_{max} is max electron energy

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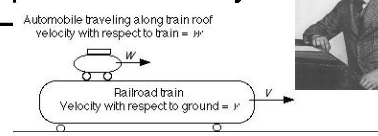
Special Relativity



- Einstein modified Galilean relativity, under which velocities are additive.
- Galileo: automobile velocity with respect to ground = $u = v + w$
- Einstein says: $u < c$ so we need new rules!

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Special Relativity: Energy

Hendrik Lorentz

- Einstein's new rules require that *time* & *distance* formulae depend on velocity
- Corrections are significant in nuclear reactions, radiation scattering, and accelerators, so we study them here a little
- They also give rise to the concept of relativistic energy:
  $E = m_0c^2/\sqrt{(1-v^2/c^2)}$, or
  $E = \Upsilon m_0c^2$, where $\Upsilon = (1-v^2/c^2)^{-1/2}$

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Rest energy and mass

- If $v = 0$, $E = E_0 = m_0c^2$; this is *rest energy* if m_0 is the rest mass, i.e. the mass as it is ordinarily defined.
- We can summarize the results by defining a relativistic mass m so that we can say
  $E = mc^2 = \gamma m_0c^2$ where $m = \gamma m_0$
- For $v = 0.1c$, $m = 1.05m_0$;
  for $v = 0.98c$, $m = 5m_0$.

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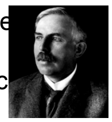
Atomic Structure

- JJ Thomson (1897): heavy nucleus with electrons surrounding it.



- Rutherford showed that the nucleus had to be very small relative to the atomic size

The Bohr Model

- Bohr model: quantized angular momentum so that radiation is emitted in quanta equal to difference between energy levels of the atom. Used classical energy calculations!

Niels Bohr

Bohr model: radius

- Quantized angular momentum $mvr = nh/2\pi$
- But this is associated with coulombic attraction for which the centripetal force must equal the coulombic force:
  $F = mv^2/r = kZe^2/r^2$, so $r = kZe^2/mv^2$
- Thus $v = nh/(2\pi mr) = 2\pi kZe^2/nh$
  so $r = n^2h^2/(4\pi^2kZe^2m)$
- For $n=Z=1$, $r = 0.529*10^{-10}$ m = Bohr radius

Bohr model: Electron Energies

- Velocity = $v = 2\pi kZe^2/(nh)$
- Kinetic energy = $1/2mv^2 = 2\pi^2k^2Z^2e^4m/(n^2h^2)$
- Potential energy = $-kZe^2/r = -4\pi^2k^2Z^2e^4m/(n^2h^2)$
- Total energy = KE + PE = $-2\pi^2k^2Z^2e^4m/(n^2h^2)$
- Photons emerge from transitions from one value of n to another.
- Transition from n = 3 to n = 2 gives photon energy = $-2\pi^2k^2Z^2e^4m$ / [(1/9 - 1/4) h^2] = 1.89 eV

DeBroglie Wave Theory

Louis-Victor, 7th duc de Broglie; photo courtesy *Encyclopædia Brittanica*

- So: we've allowed electromagnetic radiation to behave as a wave and a particle. We can express momentum of light as
  $P = E/c = h\nu/c = h/\lambda$
- Can we also talk about matter behaving both as a wave and a particle? *Yes.*
- Particles can exhibit interference effects associated with wave behavior.
- Wavelength $\lambda = h/P = h / mv$

Wave Behavior in electrons

- Nonrelativistic approximation:
  $KE = (1/2)mv^2$ so $\lambda = h/(mv) = h(2(KE)m)^{-1/2}$
- Further, since the angular momentum mvr is quantized ($mvr = nh/(2\pi)$), we can say
  $2\pi r = n\lambda$
- So we can say that the circumference of the electron's orbit is an integer multiple of the electron's wavelength! Standing waves!

Assignment associated with this lecture:

- Alpen, chapter 2, problem 1:
  Assume an oscillating spring that has a spring constant, *k*, of 20 Nm^{-1}, a mass of 1 kg, and an amplitude of 1 cm. If Planck's radiation formula describes the behavior of this system, what is the quantum number, n. What is ΔE if *n* changes by 1? The frequency of a simple oscillator is given by
  $\nu = (1/2\pi)(k/m)^{1/2}$

Assignment, continued:

- Alpen, ch.2, problem 4:
  A proposed surface for a photoelectric light detector has a work function of $2.0*10^{-19}$ J. What is the minimum frequency of radiation that it will detect? What will be the maximum kinetic energy of electrons ejected from the surface when it is irradiated with light at 3550Å (355 nm)?

Assignment, continued:

- Alpen, chapter 2, problem 5:
  In the previous problem, what is the de Broglie wavelength of the maximum kinetic energy electron emitted from the surface? What is its momentum?

Assignment, concluded:

- (from my head…):
  The Advanced Photon Source (APS) at Argonne National Laboratory produces X-rays from electrons that have been accelerated to an energy of approximately 7 gigaelectron volts. This corresponds to an electron velocity very close to the speed of light. If an APS electron's speed is *v*, calculate *c*-*v* in m/sec to two significant figures.