Illinois Institute of Technology

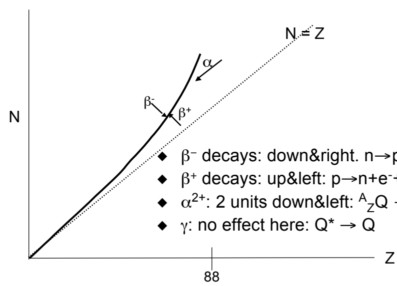
RADIATION BIOPHYSICS
Lecture 2a: Radioactivity

ANDREW HOWARD

Stable and Unstable Elements

- Every element has ≥1 unstable isotope, i.e. one that undergoes radioactive decay
- Most elements with $Z < 92$ have at least one stable isotope
- We'll examine radioactivity in terms of the transitions under which an atom decays
- Radioactivity has various influences on biological tissue:
  - Ionization of biological macromolecules
  - Indirect effects, often via free radicals
  - Medical applications: therapy, diagnostics, . . .

Region of stability and nuclear decay



- β^- decays: down&right. $n \to p + e^- + \bar{\nu}$
- β^+ decays: up&left: $p \to n + e^- + \nu$
- α^{2+}: 2 units down&left: ${}^{A}_{Z}Q \to {}^{A-4}_{Z-2}R$
- γ: no effect here: $Q^* \to Q$

Radioactivity

- Nuclear Stability
- Mass Decrement
- Alpha Emission
- Negative Beta Emission
- Positive Beta Emission
- Electron Capture
- Spontaneous Fission

Rules

- Stable nuclei of even Z more numerous than odd Z.
- Stable nuclei of even N more numerous than odd N.
- Stable nuclei of even A more numerous than odd A.
- In general, stable nuclei of even A have even Z. Some exceptions exist, however, such as ${}^{2}H$, ${}^{6}Li$, ${}^{10}B$, and ${}^{14}N$
- Only two stable structures are known for which Z is greater than N: ${}^{3}He$ and ${}^{1}H$
- Note what happens to the N/Z ratio in a typical alpha decay:

$${}^{226}Ra \to {}^{222}Rn + \alpha + \Upsilon + Q$$

Z = 88	86	(number of protons)
N = 138	136	(number of neutrons)
N/Z = 1.5682	1.5814	

Mass Energy of α - Particle

Ignoring binding energy

$2 * m_o c^2$(neutron) ≈ 1978 MeV
$2 * m_o c^2$(proton) ≈ 1976 MeV

Mass Energy = 3954 MeV
Kinetic Energy ~ 4 MeV

Beta Decays

Negative Electron Decay
$^{A}X \rightarrow {}^{A}Y + \beta^- + -\nu + Q$ (Z increases by 1: $n \rightarrow p$)

Positive Electron Decay
$^{A}X \rightarrow {}^{A}Y + \beta^+ + \nu + Q$ (Z decreases by 1: $p \rightarrow n$)

Spontaneous annihilation

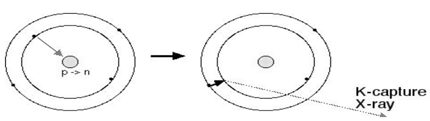
β^+ +	e^-	$\rightarrow 2\Upsilon$
0.511 MeV +	0.511 MeV	$\rightarrow$ 1.022 MeV

Energy - Wavelength Relationship for Photons

- The general relation connecting wavelength to energy is E = hc / λ
- Specifically, for energy in eV and wavelength in Å: E = 12398.4 / λ
- For energy in MeV and wavelength in Å: E = 0.0123984 / λ
- for the photons emitted in a positron annihilation:
  E = 0.511 MeV so λ = 0.0123984 / 0.511
  = 0.02426 Å = 2.426 pm.

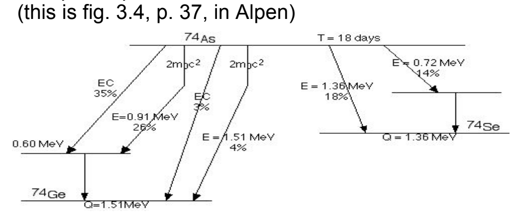
Electron Capture

Neutron-deficient species can capture an electron from an inner shell of the atom. Unlike conventional positron decays, for which the energy difference Q must be at least
$2m_0c^2$ = 0.511 MeV,
the electron-capture process has no minimum energy requirement.



Charting Decay Schemes

We can sometimes find multiple pathways, each with multiple steps, as with ^{74}As here
(this is fig. 3.4, p. 37, in Alpen)



Nomenclature for Nuclei

Incorporates
- Atomic Number
- Atomic Mass Number
- Neutron Number (implicitly)

$^{A}{}_{Z}$*Chemical Symbol*

Simplified form:
^{A}*Chemical Symbol*

Example:

^{238}U

Law of Radioactivity

- Rate of disappearance is proportional to the quantity of original nuclide remaining. This is an intuitively straightforward notion, but it gives rise to a simple differential equation relating disappearance of atoms, d*N*/d*t*, to the number of atoms present, *N*:
- $dN/dt \propto N$
- Therefore $dN/dt = -\lambda N$
- This equation, together with knowing that *N* is a known value N_0 at time t=0, is all we need to write down the law of radioactivity!

Radioactivity: differential equation

- This can be rewritten $dN/N = -\lambda dt$, which is a simple first-order linear differential equation.
- Solution is $\ln N = -\lambda t + Q$. Exponentiating both sides, $N(t) = \exp(-\lambda t)\exp(Q)$.
- Cauchy boundary condition is $N = N_0$ at $t = 0$; therefore $N_0 = N(0) = \exp(-\lambda * 0)\exp(Q) = \exp(Q)$ so (ta da!) $N(t) = N_0\exp(-\lambda t)$

Half-life and activity

- Half-life ≡ time at which $N = N_o/2$
- So the condition we place on the number N is $N = N_o/2$
- $N = N_o/2 = N_o \exp(-\lambda t_{1/2})$
- Therefore $1/2 = \exp(-\lambda t_{1/2})$
- Taking the ln of both sides, $\ln(1/2) = -\lambda t_{1/2}$
- $-\ln(2) = -\lambda t_{1/2}$, i.e. $t_{1/2} = \ln(2) / \lambda = 0.693 / \lambda$
- This provides the relationship between $t_{1/2}$ and the activity coefficient λ
- The *mean life* is defined as $1/\lambda$ or $1.443\ t_{1/2}$

Decay of mixtures

- Suppose we have several nuclides present in the same sample.
- The most common circumstance of this kind involves an emitter that decays into something else that decays further, but it doesn't have to be that way.
- Total activity is the sum of the activities of the individual nuclides: $A_{total} = A_1 + A_2 + A_3 + \ldots$ $= \lambda_1 N_1 + \lambda_2 N_2 + \lambda_3 N_3 + \ldots$

Chain Decay

- Take simple case where A decays to B, and B decays to C, which is stable.
- The activity coefficients are λ_1 and λ_2.
- At time t=0:
  - Activity of parent = $\lambda_p N_p$
  - Activity of daughter = $\lambda_d N_d$
- Then $dN_p/dt = -\lambda_p N_p$ and $dN_d/dt = -\lambda_d N_d + \lambda_p N_p$.
- This can be integrated to yield $N_d(t) = N_p(t_0)(\lambda_p/(\lambda_d - \lambda_p))(\exp(-\lambda_p t) - \exp(-\lambda_d t)) + N_d(t_0)\exp(-\lambda_d t)$
- Activity: $A_d(t) = A_p(t_0)(\lambda_d/(\lambda_d - \lambda_p))(\exp(-\lambda_p t) - \exp(-\lambda_d t)) + A_d(t_0)\exp(-\lambda_d t)$

Special case I: $\lambda_p << \lambda_d$

- *Secular equlibrium*: $\lambda_p << \lambda_d$, I.e. $T_p >> T_d$:
- Activity is essentially constant over several half-lives of the daughter T_d.
- In the equation $\lambda_p << \lambda_d$ so $\lambda_d/(\lambda_d - \lambda_p) \sim 1$
- Therefore if $A_d(t_0) = 0$, then $A_d(t) = A_p(t_0)(1 - \exp(-\lambda_d t))$
- After several half-lives the last term here will be zero: $A_d(t) = A_p(t_0)$
- So the activity of the daughter is determined solely by the decay constant of the parent

Special case 2: $\lambda_p < \lambda_d$, but close

- After several half-lives of the daughter, $\exp(-\lambda_p t) << \exp(-\lambda_d t)$ so (again with $A_d(t_0) = 0$),
- $A_d(t) = A_p(t_0)\lambda_d/(\lambda_d - \lambda_p)\exp(-\lambda_p t)$
- But at any time, $A_p(t) = A_p(t_0)\exp(-\lambda_p t)$, $A_d(t) = A_p(t)\lambda_d/(\lambda_d - \lambda_p)$
- We call this transient equilibrium
- Note that $A_d / A_p = \lambda_d/(\lambda_d - \lambda_p)$
- Rewriting with half-lives, $A_d / A_p = T_p/(T_p - T_d)$

Max daughter activity

- Max occurs when the general equation for $A_d(t)$ is at its maximum.
- For $A_d(t_0) = 0$, that means the maximum of $A_d(t) = A_p(t_0)(\lambda_d/(\lambda_d - \lambda_p))(\exp(-\lambda_p t) - \exp(-\lambda_d t))$
- Set the derivative of this equal to zero: $dA_d(t)/dt = 0 = A_p(t_0)(\lambda_d/(\lambda_d - \lambda_p))^*$ $(-\lambda_p \exp(-\lambda_p t) + \lambda_d \exp(-\lambda_d t))$
- Dividing through by $A_p(t_0)(\lambda_d/(\lambda_d - \lambda_p))$, $0 = -\lambda_p \exp(-\lambda_p t) + \lambda_d \exp(-\lambda_d t)$

Max daughter activity, concluded

- $\lambda_p \exp(-\lambda_p t) = \lambda_d \exp(-\lambda_d t)$
- $\lambda_p / \lambda_d = \exp(-\lambda_d t + \lambda_p t)$
- $\ln(\lambda_p / \lambda_d) = (-\lambda_d + \lambda_p)t$
- $t = \ln(\lambda_p / \lambda_d) / (-\lambda_d + \lambda_p) = 1.441\ T_p T_d/(T_d - T_p) \ln(T_d/T_p)$
- That may be more easily written $t = 1.441\ T_p T_d/(T_p - T_d) \ln(T_p/T_d)$
- Comment in text on p. 33 is wrong: $1.441\ T_p T_d$ is not the product of the mean life of the parent and the daughter.

Special Case 3: $T_p < T_d$

- Non-equilibrium: $T_p < T_d$
- No straightforward simplification of the equation occurs. However!
- For a substantial case, $T_p << T_d$, $\lambda_p >> \lambda_p$, we can make some interesting simplifications:
  - $\lambda_d/(\lambda_d - \lambda_p) \sim -\lambda_d/\lambda_p$
  - After a few of the smaller half-lives, $\exp(-\lambda_p t) - \exp(-\lambda_d t) \sim 0 - \exp(-\lambda_d t) = -\exp(-\lambda_d t)$
- So $A_d(t) = A_p(t_0)(-\lambda_d/\lambda_p)(-\exp(-\lambda_d t)) + A_d(t_0)\exp(-\lambda_d t)$
- Thus $A_d(t) = A_p(t_0)(\lambda_d/\lambda_p)\exp(-\lambda_d t)) + A_d(t_0)\exp(-\lambda_d t)$
- Finally $A_d(t) = (A_p(t_0)\lambda_d/\lambda_p + A_d(t_0))\exp(-\lambda_d t)$

Mass Defect

- For any isotope, mass defect defined as $\Delta = M - A$
- M = measured mass of isotope
- A = mass number, i.e. count of nucleons
- Sounds good; but definition of atomic mass units is arbitrary. The correction is based on forcing D=0 for oxygen-16 (physicists) or carbon-12 (chemists)
- That means this isn't really useful.

Mass decrement

- Mass decrement defined as $\delta = W - M$
  - W = Sum of masses of nucleons in nucleus
  - M = Measured mass of nucleus
- This can be defined in amu or in ordinary units like MeV or even kg
- This one is actually useful in estimating available energy in determining whether an isotope is stable and in estimating available energies of various α and β decays

Example of mass decrement

- ^{4}He is highly stable.
- W = 2*1.007276+2*1.008650+2*0.0005486=4.032949 amu
- Measured M = 4.00260 so δ = W-M = 0.030349 amu
- That corresponds to 28.27 MeV
  - ~3% of the rest energy of a nucleon
  - ~55.3 * the rest energy of an electron
- That's the energy that would be released if 2 protons, 2 neutrons, and 2 electrons were brought together to form a helium atom
- Mass decrement doesn't, by itself, serve as a predictor of stability. But it helps.

Beta decay for ^{41}Ar to ^{41}K

- Product's isotopic mass $M(^{41}K)$ = 40.9784 amu
- Starting isotopic mass is $M(^{41}Ar)$ = 40.98108 amu
- Difference in δ is therefore 0.00268 amu
- This is spread between the β and the γ
- β is 0.00129 amu and γ is 0.00139 amu.