Illinois Institute of Technology

RADIATION BIOPHYSICS
Lecture 4:
Interaction of Photons with Matter

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Exposure: A More Detailed Definition

Exposure is defined as *charge to which the target material is exposed per unit mass*

Exposure = $\Delta Q/\Delta m$

The unit of exposure to electromagnetic radiation is the *Roentgen*

The Roentgen was defined as 1 esu/cm^3 air at STP

$\rho(\text{air})_{STP} = 1.293 \cdot 10^{-3}$ g/cm^3

1 esu = $3.34 \cdot 10^{-10}$ Coulomb

So 1R = $3.34 \cdot 10^{-10}$ C /(1 cm^3 * $1.293 \cdot 10^{-6}$ kg/cm^3)

i.e. 1 Roentgen = $2.58 \cdot 10^{-4}$ C/kg

Some Radioactivity Problems

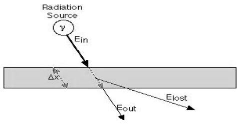
1. Chapter 3, problem 1:

The atomic mass, m, of ^{64}Cu is 63.929757 amu. It undergoes positron decay with a half-life of 12.9 h. The product of this decay is ^{64}Ni. The mass, m, of this product is 63.927956. What is the total energy of the positron and the neutrino resulting from the decay? Is the product liable to be stable or be radioactive? Why?

2. Chapter 3, problem 3

A source of ^{99m}Tc arrives at the laboratory for use at 10 AM on a Monday morning, at which time this daughter product is eluted for diagnostic use. The parent, ^{99}Mo, has a decay constant of 0.01039 h^{-1}. If, after the separation of the daughter, the parent was found to have an activity of $5.0 \cdot 10^9$ Bq, what is the activity of the parent and the daughter the following Thursday at 10 am?

Interactions of Photons with Absorber



Number of photons not transmitted:

$\Delta N = \mu \Delta x N$

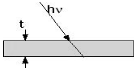
μ is the *linear attenuation coefficient*

Units of μ: inverse length

Energies: E_{in}-E_{out} = $E_{transferred}$

$E_{absorbed}$ = amount that stays = (E_{in}-E_{out}) - E_{lost}

Interaction of Photons with Matter



- Let N_0 = Number of photons in
- N = Number of photons out
- t = thickness of absorber
- μ = attenuation coefficient (dimensions: L^{-1})
- Then: $N = N_0 exp(-\mu t)$
- Mass attenuation coefficient μ/ρ (ρ = density)
- Since dimensions of density are ML^{-3}, μ/ρ has dimensions of L^{2}M^{-1}; it's a *cross-section*

Cross Section

- We keep finding quantities that look like L^2 in the numerators of these definitions, so physicists like to describe these as cross-sections, i.e. we think about the effective cross-sectional area of the electron as it interacts with the photon.
- $\tau/\rho \propto E^{-3}$ of the photon (most elements),
- $\tau/\rho \propto E^{-2.96}$ of the photon (for lead)
- But energy = hc/λ, so for most elements $\tau/\rho \propto \lambda^3$

Cross-Section and Attenuation

Attenuations described in terms of L^2/(something)

Area → cross section

large cross section ⇒ high probability of interaction

Thus several kinds of attenuation coefficients:

Type	Dimensions	Units
◆ Linear(μ)	L^{-1}	m^{-1}
◆ Mass (μ/ρ)	L^2M^{-1}	m^2kg^{-1}
◆ Electronic ($_e\mu$)	L^2q^{-1}	m^2e^{-1}
◆ Atomic ($_a\mu$)	$L^2(\text{atom})^{-1}$	$m^2(\text{atom})^{-1}$

Energy Transferred and Absorbed from Photons by Carbon:

Table 4.2 in book:
Energy Transferred and Energy Absorbed for Incident Photons of Various Energies
(for Carbon)

Photon Energy, E_{tot} MeV	Average Energy Transferred, E_{tr} MeV	Average Energy Absorbed, E_{ab} MeV
0.01	0.00865	0.00865
0.10	0.0141	0.0141
1.00	0.440	0.440
10.0	7.30	7.04
100.0	95.6	71.90

Mechanisms of Energy Transfer

Gamma Rays

- Photoelectric scattering:

  Photoelectron KE = $h\nu$ - binding energy
  - Must involve bound electrons
  - Partial cross section $\tau/\rho \propto (h\nu)^u$, roughly:

    u = -3 for low-Z elements

    u = -2.96 for Pb

Energy Transferred and Absorbed

Energy in, out, absorbed, and leaving:

$E_{in} \rightarrow E_{tr} + E_{out}$

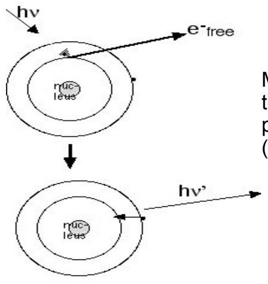
$E_{tr} = E_{abs} + E_{leave}$

so transferred energy is greater than absorbed energy

We define separate attenuation coefficients:

- Energy transfer attenuation coefficient
- Energy absorbed attenuation coefficient

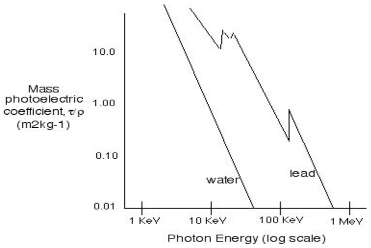
Photoelectric Effect



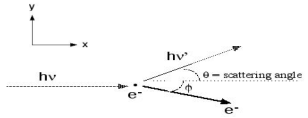
Most significant at low to intermediate photon energies
(~ 10 - 100 keV)

Cross-section falls off rapidly with Energy and is Z-dependent

K and L orbital edges fall within the plot for many metals, not for light atoms



Compton Scattering



1. Energy conservation
2. Momentum conservation in x
3. Momentum conservation in y

Energy conservation:

$E_{e^-,rest} + E_{\gamma,in} = E_{\gamma,out} + E_{e^-,out}$

$m_0c^2 + h\nu = h\nu' + \Upsilon m_0c^2$

Momentum Conservation in x:

$h\nu/c = (h\nu'/c)\cos\theta + \Upsilon m_0 v\cos\phi$

Momentum conservation in y:

$0 = (h\nu'/c)\sin\theta - \Upsilon m_0 v\cos\phi$

Compton Scattering: Equations

- For convenience we define $\alpha = h\nu / m_0c^2$
- This is a unitless measure of photon energy
- Since $m_0c^2 = 0.511$ MeV, if $h\nu = 5.11$ MeV, then $\alpha = 10$.
- This enables us to use these three equations (in energy, momentum in X, and momentum in y) to solve for three variables, given a known value of a
- Those three unknowns are θ, ϕ, and ν'.

Compton Scattering: Results

- $KE^{(e)}_{out} = h\nu\alpha(1-\cos\theta)/(1+\alpha(1-\cos\theta))$
- $h\nu' = h\nu - KE^{(e)}_{out}$
- $h\nu' = h\nu - h\nu(\alpha(1-\cos\theta))/(1+\alpha(1-\cos\theta))$
- $h\nu' = h\nu[1+\alpha(1-\cos\theta) - \alpha(1-\cos\theta)]/(1+\alpha(1-\cos\theta))$
- $h\nu' = h\nu / (1+\alpha(1-\cos\theta))$

Compton Scattering: Special Cases

- Recoil: $\theta = 180°$, $\cos\theta = -1$
  $KE^e_{out} = 2h\nu\alpha / (1 + 2\alpha)$;
  $h\nu' = h\nu/(1 + 2\alpha)$; max. energy to e^-.
- Minimum energy transfer to electron:
  $\theta = 0°$, $\cos\theta = 1$, $KE^e_{out} \approx 0$, $h\nu' = h\nu$
- Low-energy photon, $\alpha << 1$:
  $h\nu' \approx h\nu$, $KE^e_{out} \approx h\nu\, \alpha(1-\cos\theta)$
- High-energy photon, $\alpha >> 1$:
  $h\nu' \approx h\nu / [\alpha(1-\cos\theta)]$, $KE^e_{out} \approx h\nu$
  (minor significance)

Compton Cross Section: Klein-Nishina formulation

diff: [illegible]

Thompson scatt cross section

Thompson differential x - section

r_o = classical electron radius = $2.8 \cdot 10^{-15}$ M

total cross [illegible]

[illegible] where k is coulomb-law constant

total cross-section for Thompson case:

[illegible]

Formulae for Cross-Sections

Total Cross-section:

[illegible]

Compton Scattering Cross Section

[illegible]

[illegible]

F_{KN} is a geometry-dependent quantum-mechanical factor given as eqn. 4.30;

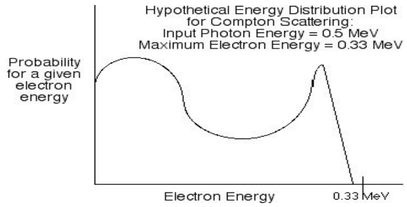
$F_{KN} \leq 1$; $F_{KN} \rightarrow 1$ as $\theta \rightarrow 0°$ or as $\alpha \rightarrow 0$.

Energy of Compton Electrons

- From standard Klein-Nishina equations we can determine the spectrum of Compton electon energies.
- KE_{max} of electron is close to photon energy
  - E_γ = 0.5 MeV implies $KE_{max}(e^-)$ = 0.331 MeV
  - E_γ = 1.0 MeV implies $KE_{max}(e^-)$ = 0.796 MeV

Distribution of Energies

Energy distributions go through a minimum: low- and high-energy electrons are more common than intermediate-energy electrons



Effect of Binding Energy

- Typical Compton treatments assume free electrons: this is close to right.
- Sharp fall-off in total coefficient at low energies (below 50 KeV), but not much gets transferred at those energies anyway, it doesn't affect the equations much

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Pair Production

- Can happen if E_γ > 1.022 MeV = 2 * $m_0(e^-)$
- Rapidly increasing cross-section > 1.022 MeV
- Stopping power/atom varies as Z^2
- Energy transferred is ($h\nu$ - 1.022) MeV
- Scattering nucleus plays fairly passive role (not much momentum transferred to nucleus)
- Generally the positron gets annihilated, giving off another pair of 0.511 MeV photons. These generally escape and are not part of the absorbed energy

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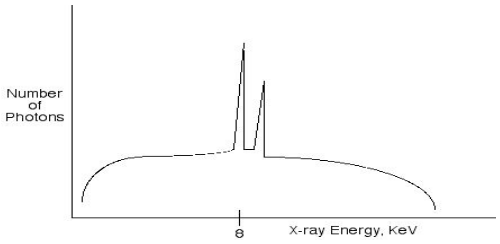
Bremsstrahlung: Radiative Energy Loss

- "Braking radiation":
  A fast electron loses energy to its environment in a nonspecific way due to Coulombic interaction with neighboring charged particles.
- The static particles are much more massive than the electron, so they don't get accelerated nearly as much as the electron does: but the electron does get accelerated.
- What happens when an electron is accelerated? It has to radiate! This type of Coulombically-motivated radiation is *Bremsstrahlung*

Significance of Bremsstrahlung

- Example in X-ray generators:
- 1.5418Å (8KeV) X-rays are produced in great quantity when we shoot fast electrons at a copper target
- BUT: we also get a lot of radiative transfer of energy from the electrons as the move past the copper atoms. This gives rise to Bremsstrahlung, which has no characteristic energies.
- Thus the spectrum is like this:

Output X-ray Spectrum of a Copper Target



Energy Transfer

- Compton Processes in Tissue
- Charged Particles and Matter
- Final Steps in Energy Absorption
- Dose and Kerma: A Review
- Neutron Interactions with matter

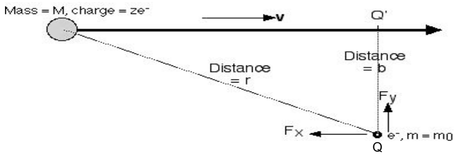
Compton Processes in Tissue

Biological soft tissue is predominantly made up of H, C, N, O, and a little P and S. So attenuation of photons is dominated by those light elements ($Z \leq 16$)

Interaction of Charged Particles with Matter

See pages 84 & 85 in the text-

Provides solutions to the dynamical equations describing motion of a heavy charged particle past a stationary electron or (by relativity) motion of an electron past a stationary heavy particle: $F = kze^2/r^2$ along line MQ



Interaction of e^- With Heavy Charged Particle

[illegible]

[illegible]

- Momentum imparted to electron associated with force:

[illegible]

classical coherent

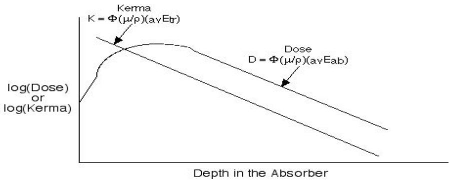
[illegible]

non relativistic [illegible]

Dose and Kerma

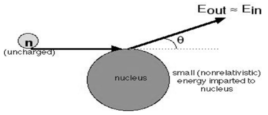
See Fig. 5.5 in text.

Because secondary events extend farther into tissue (or other) than the initial deposited radiation, dose extends farther into the interior than kerma.



Neutrons: Elastic Scatter

Important up to ~14 MeV range



Energy imparted to nucleus:

[illegible]

average over angles:

[illegible]

[illegible]

How to average $\cos^2\theta$:

- Addition formula for cosine:
  - $\cos(A+B) = \cos A \cos B - \sin A \sin B$
  - For $A=B=\theta$, $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
  - Furthermore $\cos^2\theta + \sin^2\theta = 1$ so
  - $\cos 2\theta = \cos^2\theta - (1 - \cos^2\theta) = 2\cos^2\theta - 1$
  - Therefore $\cos^2 q = (1 + \cos 2\theta) / 2$
- This gives us the tools we need to integrate $\cos^2\theta$ over an interval.
- In general <f(x)> over an interval (a,b) is

[illegible]

$\langle\cos^2\theta\rangle$, continued

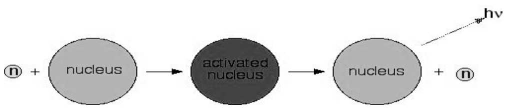
[illegible]

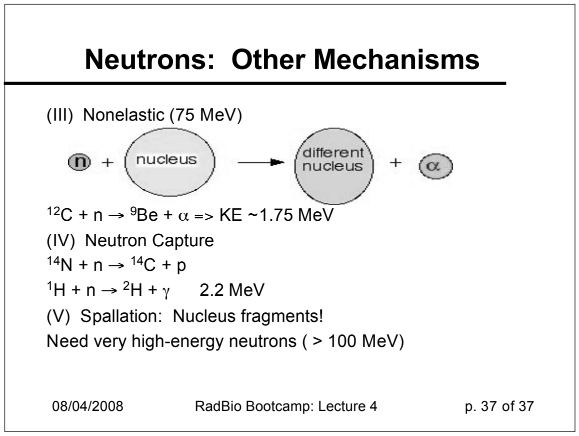
Significance in Elastic Scatter

- Recall we said that for any value of q, the energy transferred to the target nucleus, Et, is $E_t = E_n (4M_aM_n)\cos^2\theta / (M_a + M_n)^2$
- So the average energy imparted to the target nucleus is
- $\langle Et \rangle = \{E_n (4M_aM_n) / (M_a + M_n)^2\} \langle\cos^2\theta\rangle$
- We just spent three pages proving $\langle\cos^2\theta\rangle = 1/2$
- Thus $\langle Et \rangle = 2E_nM_aM_n / (M_a + M_n)^2$

Inelastic Scatter

Increasingly important at higher neutron energies





Neutrons: Other Mechanisms

(III) Nonelastic (75 MeV)

$^{12}C + n \rightarrow {}^{9}Be + \alpha \Rightarrow$ KE ~1.75 MeV

(IV) Neutron Capture

$^{14}N + n \rightarrow {}^{14}C + p$

$^{1}H + n \rightarrow {}^{2}H + \gamma$ 2.2 MeV

(V) Spallation: Nucleus fragments!

Need very high-energy neutrons (> 100 MeV)