Illinois Institute of Technology

PHYSICS 561
RADIATION BIOPHYSICS

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Housekeeping

The homework associated with last week's lecture is due at 11:59 pm Friday 2 Feb. Homework for the internet students is due one week later. This pattern will persist throughout the semester. Remember that the course's internal website is http://icarus.csrri.iit.edu/radbio/

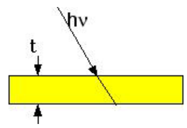
and the material there is updated frequently.

Homework may be turned in either on paper, by e-mail, or by fax. Faxes for Prof. Howard:
630-252-0521

312-567-3576

Prof. Howard's pager number is 312-902-9816.

Interaction of Photons with Matter



- Let N_0 = Number of photons in
- N = Number of photons out
- t = thickness of absorber
- μ = attenuation coefficient (dimensions: L^{-1})
- Then: $N = N_0 exp(- \mu t)$
- Mass attenuation coefficient μ/ρ (ρ = density)
- Since dimensions of density are ML^{-3}, μ/ρ has dimensions of L^2M^{-1}; it's a *cross-section*

Cross-Section and Attenuation

Attenuations described in terms of L^2/(something)

Area → cross section

large cross section ⇒ high probability of interaction

Thus several kinds of attenuation coefficients:

Type	*Dimensions*	*Units*
Linear(μ)	L^{-1}	m^{-1}
Mass (μ/r)	L^2M^{-1}	m^2kg^{-1}
Electronic ($_e\mu$)	L^2q^{-1}	m^2e^{-1}
Atomic ($_a\mu$)	$L^2(atom)^{-1}$	$m^2(atom)^{-1}$

Energy Transferred and Absorbed

Energy in, out, absorbed, and leaving:

$E_{in} \rightarrow E_{tr} + E_{out}$

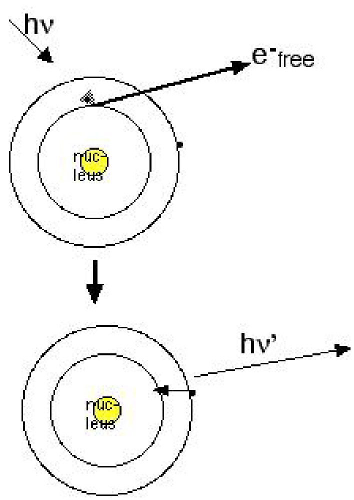
$E_{tr} = E_{abs} + E_{leave}$

so transferred energy is greater than absorbed energy

We define separate attenuation coefficients:

- Energy transfer attenuation coefficient
- Energy absorbed attenuation coefficient

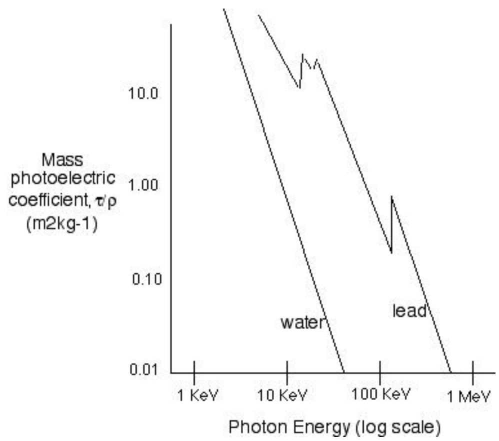
Photoelectric Effect



Most significant at low to intermediate photon energies (~ 10 - 100 keV)

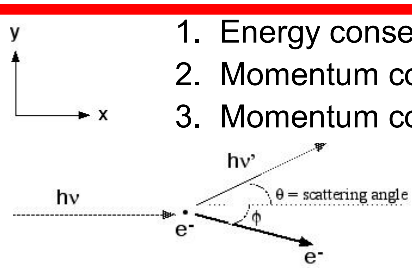
Cross-section falls off rapidly with Energy and is Z-dependent

K and L orbital edges fall within the plot for many metals, not for light atoms



Compton Scattering

1. Energy conservation
2. Momentum conservation in x
3. Momentum conservation in y



Energy conservation:

$E_{e^-,rest} + E_{\gamma,in} = E_{\gamma,out} + E_{e^-,out}$

$$m_o c^2 + h\nu = h\nu' + \frac{m_o c^2}{\sqrt{1 - v^2/c^2}}$$

Momentum conservation in x and y:

$$\frac{h\nu}{c}\hat{x} = \frac{h\nu'}{c}(\hat{x}\cos\theta + \hat{y}\sin\theta) + \frac{m_o v}{\sqrt{1 - v^2/c^2}}[\cos\phi\hat{x} - \sin\phi\hat{y}]$$

Compton Scattering: Equations

$$\hat{x}: \quad \frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + \frac{m_o v}{\sqrt{1 - v^2/c^2}}\cos\phi$$

$$\hat{y}: \quad \frac{h\nu'}{c}\sin\theta - \frac{m_o v}{\sqrt{1 - v^2/c^2}}\sin\phi$$

$$Energy: \quad h\nu - h\nu' = \frac{1}{\sqrt{1 - v^2/c^2}} - 1$$

$$\rightarrow \alpha = \left[\frac{h\nu}{m_o c^2}\right]$$

Since m_0c^2 = 0.511 MeV,
if hν = 5.11 MeV, then α = 10.

Compton Scattering: Results

$$KE_{out}^{(e)} = \frac{h\nu\alpha(1-\cos\theta)}{1+\alpha(1-\cos\theta)}$$

$$h\nu' = h\nu - KE_{out}^{(e)}$$

$$h\nu' = h\nu - h\nu\left(\frac{\alpha(1-\cos\theta)}{1+\alpha(1-\cos\theta)}\right)$$

$$= h\nu\left[\frac{1+\alpha(1-\cos\theta) - a(1-\cos\theta)}{1+\alpha(1-\cos\theta)}\right]$$

$h\nu' = h\nu / (1 + \alpha (1 - \cos\theta))$

Compton Scattering: Special Cases

- Recoil: $\theta = 180°$, $\cos\theta = -1$
  $KE^e_{out} = 2h\nu\alpha / (1 + 2\alpha)$;
  $h\nu' = h\nu/(1 + 2\alpha)$; max. energy to e^-.
- Minimum energy transfer to electron:
  $\theta = 0°$, $\cos\theta = 1$, $KE^e_{out} \approx 0$, $h\nu' = h\nu$
- Low-energy photon, $\alpha << 1$:
  $h\nu' \approx h\nu$, $KE^e_{out} \approx h\nu\, \alpha(1-\cos\theta)$
- High-energy photon, $\alpha >> 1$:
  $h\nu' \approx h\nu / [\alpha(1-\cos\theta)]$, $KE^e_{out} \approx h\nu$
  (minor significance)

Compton Cross Section: Klein-Nishina formulation

diff: $\frac{d\sigma}{d\Omega} = \frac{d\sigma_o}{d\Omega} F_{KN}$ where $\frac{d\sigma_o}{d\Omega} = \frac{r_o^2}{2}\left(1+\cos^2\theta\right)$

Thompson differential x - section

Thompson scatt cross section

r_o = classical electron radius = $2.8 \cdot 10^{-15}$ M

total cross $\Rightarrow \int_{\substack{solid\\angle}} \frac{d\sigma}{d\Omega} d\Omega$

$= \frac{ke^2}{m_o c^2}$ where k is coulomb-law constant

total cross-section for Thompson case:

$$_{Tot}\sigma_o = \int_{\substack{all\\solid\\angle}} \frac{d\sigma_o}{d\Omega} d\Omega = \frac{r_o^2}{2} \int_0^{\pi}\int_0^{2\pi} \left(1+\cos^2\theta\right)\sin\theta\, d\phi\, d\theta$$

$$= \frac{r_o^2}{2} 2\pi \int_0^{\pi} \left(1+\cos^2\theta\right)\sin\theta\, d\theta$$

$$= \pi r_o^2 \int_0^{\pi} \left(1+\cos^2\theta\right)\sin\theta\, d\theta \Rightarrow \begin{matrix} u = \cos\theta \\ du = -\sin\theta\, d\theta \end{matrix}$$

Formulae for Cross-Sections

Total Cross-section:

$$_{Tot}\sigma_o = \pi r_o^2 \int_1^{-1} (1+u^2)(-du)$$

$$= -\pi r_o^2 \left(u + \frac{u^3}{3} \right) \Big|_1^{-1}$$

$$= -\pi r_o^2 \left[-1 - 1 + \left[\left(-\frac{1}{3} \right) - \left(\frac{1}{3} \right) \right] \right]$$

$$_{Tot}\sigma_o = -\pi r_o^2 \left[-2 - \frac{2}{3} \right] = \pi r_o^2 \left(2 + \frac{2}{3} \right) = \frac{8\pi r_o^2}{3}$$

$$_{Tot}\sigma_o = \frac{2}{3} (SA)_{r_o}, \ where \quad S.A. = surface \quad area$$

$$of \quad sphere \quad 4\pi r_o^2$$

Compton Scattering Cross Section

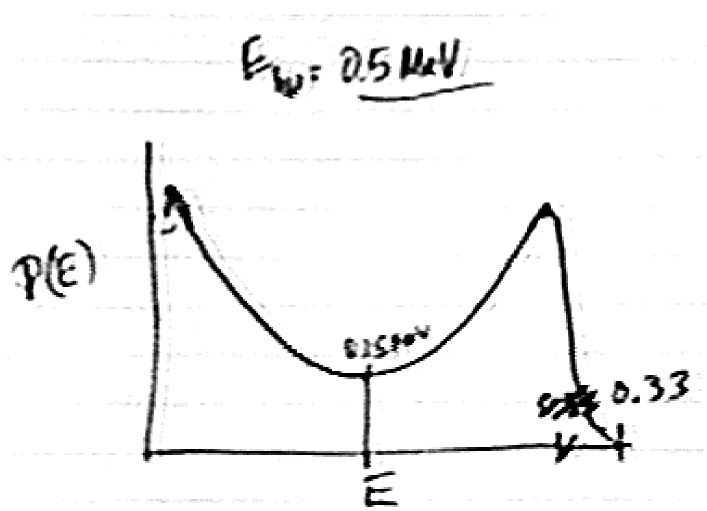
$$4.27: \frac{d_e\sigma_l}{d\Omega} = \frac{r_0^2}{2}(1+\cos^2\theta)$$

$$4.29: \frac{d\sigma}{d\Omega} = \frac{d\sigma_o}{d\Omega} F_{KN} = \frac{r_o^2}{2}(1+\cos^2\theta)F_{KN}$$

F_{KN} is a geometry-dependent quantum-mechanical factor given as eqn. 4.30;
$F_{KN} \leq 1$; $F_{KN} \to 1$ as $\theta \to 0°$ or as $\alpha \to 0$.

Energy of Compton Electrons

- From standard Klein-Nishina equations we can determine the spectrum of Compton electon energies.
- KE_{max} of electron is close to photon energy
  - E_γ = 0.5 MeV implies $KE_{max}(e^-)$ = 0.331 MeV
  - E_γ = 1.0 MeV implies $KE_{max}(e^-)$ = 0.796 MeV



Effect of Binding Energy

- Typical Compton treatments assume free electrons: this is close to right.
- Sharp fall-off in total coefficient at low energies (below 50 KeV), but not much gets transferred at those energies anyway, it doesn't affect the equations much

Pair Production

- Can happen if $E_\gamma > 1.022$ MeV $= 2 * m_0(e^-)$
- Rapidly increasing cross-section > 1.022 MeV
- Stopping power/atom varies as Z^2
- Energy transferred is ($h\nu$ - 1.022) MeV
- Scattering nucleus plays fairly passive role (not much momentum transferred to nucleus)
- Generally the positron gets annihilated, giving off another pair of 0.511 MeV photons. These generally escape and are not part of the absorbed energy

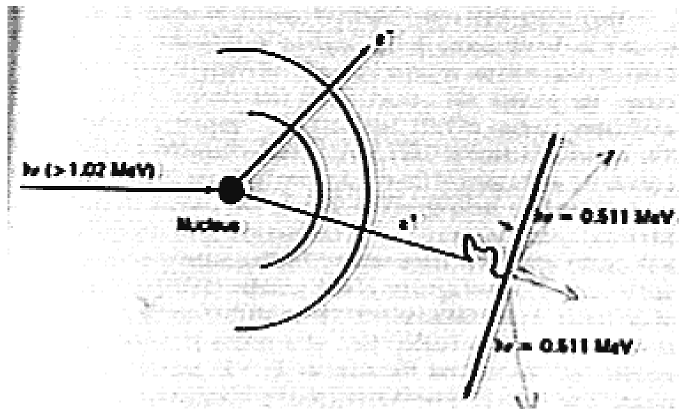


Figure 4.3 / Events in the pair production scattering process.

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Bremsstrahlung: Radiative Energy Loss

- "Braking radiation":
  A fast electron loses energy to its environment in a nonspecific way due to Coulombic interaction with neighboring charged particles.
- The static particles are much more massive than the electron, so they don't get accelerated nearly as much as the electron does: but the electron does get accelerated.
- What happens when an electron is accelerated? It has to radiate! This type of Coulombically-motivated radiation is *Bremsstrahlung*

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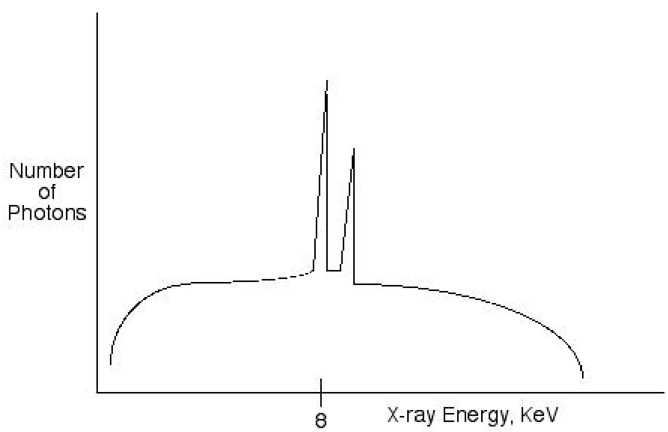
Significance of Bremsstrahlung

- Example in X-ray generators:
- 1.5418Å (8KeV) X-rays are produced in great quantity when we shoot fast electrons at a copper target
- BUT: we also get a lot of radiative transfer of energy from the electrons as the move past the copper atoms. This gives rise to Bremsstrahlung, which has no characteristic energies.
- Thus the spectrum is like this:

Output X-ray Spectrum of a Copper Target



Energy Transfer

- Compton Processes in Tissue
- Charged Particles and Matter
- Final Steps in Energy Absorption
- Dose and Kerma: A Review
- Neutron Interactions with matter

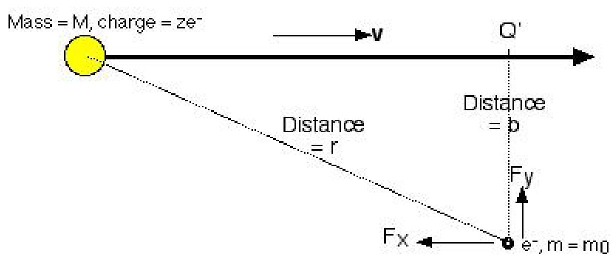
Compton Processes in Tissue

Biological soft tissue is predominantly made up of H, C, N, O, and a little P and S. So attenuation of photons is dominated by those light elements ($Z \leq 16$)

Interaction of Charged Particles with Matter

See pages 84 & 85 in the text-

Provides solutions to the dynamical equations describing motion of a heavy charged particle past a stationary electron or (by relativity) motion of an electron past a stationary heavy particle: $F = kze^2/r^2$ along line MQ



Interaction of e⁻ With Heavy Charged Particle

$$r_o = \frac{ke^2}{m_o c^2}$$

$$F = \frac{kq_M q_e}{r^2} = \frac{kze \cdot e}{r^2}$$

- Momentum imparted to electron associated with force:

$$\vec{p} = \int_{-\infty}^{\infty} \vec{F} dt$$

classical coherent

$$p_y = \int_{-\infty}^{\infty} F_y dt = zke^2 \int_{-\infty}^{\infty} \cos\theta dt = \frac{2zr_o m_o c^2}{vb}$$

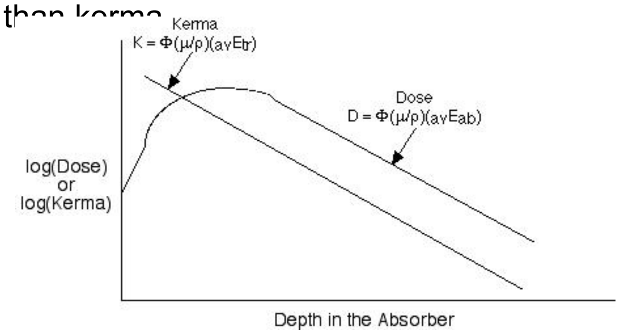
non relativistic

$$\Delta E = \frac{(\Delta p)^2}{2m_o} = \frac{z^2 r_o^2 m_o c^4 M}{b^2 E} \qquad \left(E = \frac{1}{2} M v^2\right)$$

Dose and Kerma

See Fig. 5.5 in text.

Because secondary events extend farther into tissue (or other) than the initial deposited radiation, dose extends farther into the interior than kerma.



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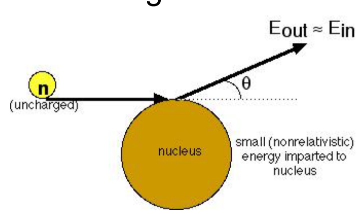
Neutrons: Elastic Scatter

Important up to ~14 MeV range

Energy imparted to nucleus:



$$E_t = E_n \frac{4M_n M_A}{(M_A + M_n)^2} \cos^2\theta$$

average over angles:

$$average\left[f(x)\right]_a^b \equiv \frac{1}{b-a}\int_a^b f(x)dx$$

$$\overline{E}_T = \frac{E_n \cdot 2M_N M_A}{(M_N + M_A)^2}$$

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average of function: $\langle f(x)\rangle_a^b = \frac{1}{b-a}\int_a^b f(x)\,dx$

$f(x) = \cos^2 x$, $\underbrace{a=0,\ b=2\pi}$

$\langle \cos^2\theta\rangle = \frac{1}{2\pi-0}\int_0^{2\pi}\cos^2\theta\,d\theta = \frac{1}{2\pi}\int_0^{2\pi}\cos^2\theta\,d\theta$.

Now $\cos^2\theta - \sin^2\theta = \cos 2\theta \leftarrow$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

in this case let $A=B=\theta$:

$\cos(\theta+\theta) = \cos 2\theta = \cos\theta\cos\theta - \sin\theta\sin\theta$

$\underline{\cos 2\theta = \cos^2\theta - \sin^2\theta}$

but $\cos^2\theta + \sin^2\theta = 1$; $\sin^2\theta = 1-\cos^2\theta$

$\cos 2\theta = \cos^2\theta - (1-\cos^2\theta)$

$\cos 2\theta = 2\cos^2\theta - 1$

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$\cos 2\theta = 2\cos^2\theta - 1$

$\frac{\cos 2\theta + 1}{2} = \cos^2\theta$

$\therefore \langle\cos^2\theta\rangle = \frac{1}{2\pi}\int_0^{2\pi}\cos^2\theta\,d\theta$

$= \frac{1}{2\pi}\int_0^{2\pi}\frac{\cos 2\theta + 1}{2}\,d\theta$

$= \frac{1}{2\pi}\left[\int_0^{2\pi}\frac{\cos 2\theta}{2}\,d\theta + \int_0^{2\pi}\frac{1}{2}\,d\theta\right]$

$\theta = \frac{1}{2}u$

let $u = 2\theta$, $du = 2d\theta$

$\frac{1}{2}du = d\theta$

$= \frac{1}{2\pi}\left[\underbrace{\int_0^{4\pi}\frac{\cos u}{2}\frac{du}{2}} + \frac{1}{2}\int_0^{2\pi}d\theta\right]$

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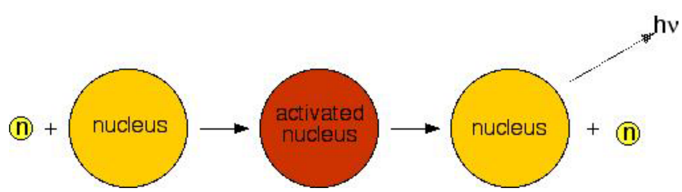
$$= \frac{1}{2\pi}\left[\frac{1}{4}\int_0^{4\pi}\cos u\,du + \frac{1}{2}\cdot 2\pi\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{4}\sin u\Big|_0^{4\pi} + \pi\right]$$

$$\langle\cos^2\theta\rangle = \frac{1}{2\pi}\left[0 + \pi\right] = \frac{1}{2}.$$

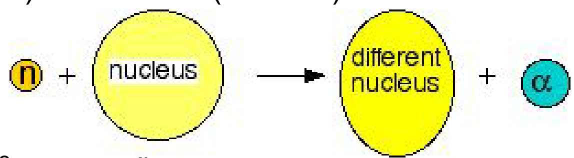
Inelastic Scatter

Increasingly important at higher neutron energies



Neutrons: Other Mechanisms

(III) Nonelastic (75 MeV)



$^{12}C + n \rightarrow {}^{9}Be + \alpha$ => KE ~1.75 MeV

(IV) Neutron Capture

$^{14}N + n \rightarrow {}^{14}C + p$

$^{1}H + n \rightarrow {}^{2}H + \gamma$ 2.2 MeV

(V) Spallation: Nucleus fragments!

Need very high-energy neutrons (> 100 MeV)

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