Illinois Institute of Technology

PHYSICS 561
RADIATION BIOPHYSICS

ANDREW HOWARD

Illinois Institute of Technology

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Second Half of First Lecture:
Electromagnetic Radiation
ANDREW HOWARD

Electromagnetic Radiation

- Much of this course deals with the interaction between electromagnetic radiation (usually ionizing) and matter
- So we need to review the properties of electromagnetic radiation
- EM radiation encompasses a wide range of energy / wavelength / frequency
  - Ionizing radiation: E = 3KeV - up, λ = 0.4 nm -down
  - Visible light is lower-energy (E~1eV, λ~500nm)

The Electromagnetic Spectrum

Category	Energy, eV	Wavelength,nm	Frequency, Hz
Radio	$10^{-10} – 10^{-5}$	$10^{8} – 10^{11}$	$2*10^{4} – 2*10^{9}$
Microwave	$10^{-5} – 10^{-2}$	10^{5} - 10^{8}	$2*10^{9} – 2*10^{12}$
Infrared	0.01-1.6	750-10^{5}	$2*10^{12} – 4*10^{14}$
Visible	1.6-3	400-750	$4*10^{14} – 7*10^{14}$
Ultraviolet	3-1000	1-400	$7*10^{14}$ - $2*10^{17}$
X-rays	10^{3}-10^{5}	10^{-2} - 1	$2*10^{17}$ - $2*10^{19}$
Gamma	10^{5}-10^{9}	10^{-6} - 10^{-2}	$2*10^{19}$ - $2*10^{23}$

Maxwell's contribution

- The four major rules of electrodynamics:
  - Coulomb's law
  - Biot-Savart law
  - Faraday's law
  - Conservation of charge
- These rules predated Maxwell. He showed they could be made self-consistent by recognizing that a changing electric field induces a magnetic field; thus the integral and differential forms of Maxwell's equations.

What Maxwell's laws mean for radiation

- The electromagnetic field travels away from its source with velocity = 3 * 10^8 m /sec. This turns out to be the velocity of light, so evidently light *is* an electromagnetic wave!
- Relationship between frequency and wavelength:
  $c = \nu\lambda$

Planck's contribution

- Planck sought to understand radiation in a cavity by assuming that the atoms in the cavity were electromagnetic oscillators with characteristic frequencies and the oscillators would absorb and emit radiation
- He also posited that the oscillators were constrained to have energies
  $E = (n+1/2)h\nu$
  where ν = frequency, h = a constant

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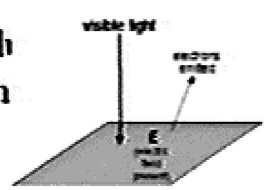
Einstein's photoelectric effect

- Energy of photons goes into enabling the electrons to escape from the surface, pl their kinetic energy after th do so:
  $E_{photon} = h\nu = E_0 + K_{max}$
- Here E_0 is the work function, K_{max} is max electron energy



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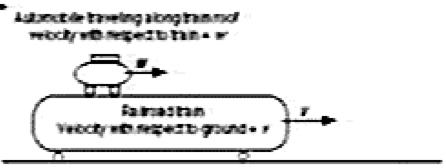
Special Relativity



- Einstein modified Galilean relativity, under which velocities are additive.
- Galileo: automobile velocity with respect to ground = $u = v + w$
- Einstein says: $u < c$ so we need new rules!

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Special Relativity: Energy

- Einstein's new rules require that *time* and *distance* formulae depend on velocity
- Corrections are significant in nuclear reactions, radiation scattering, and accelerators, so we study them here a little
- They also give rise to the concept of relativistic energy

$$E = mc^2 / \sqrt{(1 - v^2 / c^2)}, \text{ or}$$

$$E = \gamma mc^2, \text{ where } \gamma = 1 / \sqrt{(1 - v^2 / c^2)}$$

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Rest energy and mass

- If $v = 0$, $E = E_0 = m_0c^2$; this is *rest energy* if m_0 is the rest mass, i.e. the mass as it is ordinarily defined.
- We can summarize the results by defining a relativistic mass m so that we can say $E = mc^2 = \gamma m_0c^2$ where $m = \gamma m_0$
- For $v = 0.1c$, $m = 1.05m_0$; for $v = 0.98c$, $m = 5m_0$.

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-(0.1)^2}}$$

$$= \frac{1}{\sqrt{0.99}}$$

$$(1-x)^{-1/2} = 1 - \frac{1}{2}x = 1 - \frac{.01}{2}$$

$$= 1 - .005 = 0.995$$

$$x = .01$$

Atomic Structure

- JJ Thomson (1897): heavy nucleus with electrons surrounding it.
- Rutherford showed that the nucleus had to be very small relative to the atomic size
- Bohr model: quantized angular momentum so that radiation is emitted in quanta equal to difference between energy levels of the atom. Used classical energy calculations!

Bohr model: radius

- Quantized angular momentum $mvr = nh/2\pi$
- But this is associated with coulombic attraction for which the centripetal force must equal the coulombic force:

$$F = mv^2/r = kZe^2/r^2, \text{ so } r = kZe^2/mv^2$$

- Thus $v = nh/(2\pi mr) = 2\pi kZe^2/nh$

  so $r = n^2h^2/(4\pi^2 kZe^2 m)$

- For n=Z=1, r = 0.529×10^{-10} m = Bohr radius

Coulombic Force

$$F = k\frac{q_1 q_2}{r^2}$$. here one charge is an electron so $q_1 = e$

Other charge is a nucleus with Z protons, charge = Ze.

$$= \frac{k(e)(Ze)}{r^2}$$

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Bohr model: Electron Energies

- **Velocity =** $v = 2\pi k Z e^2/(nh)$
- **Kinetic energy =** $1/2mv^2 = 2\pi^2 k^2 Z^2 e^4 m/(n^2 h^2)$
- **Potential energy =** $-kZe^2/r = -4\pi^2 k^2 Z^2 e^4 m/(n^2 h^2)$
- **Total energy = KE + PE =** $-2\pi^2 k^2 Z^2 e^4 m/(n^2 h^2)$
- **Photons emerge from transitions from one value of n to another. Transition from n=3 to n=2 gives photon energy =** $-2\pi^2 k^2 Z^2 e^4 m / [(1/9 - 1/4)\, h^2]$ $= 1.89\ eV$

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DeBroglie Wave Theory

- So: we've allowed electromagnetic radiation to behave as a wave and a particle. We can express momentum of light as
  $P = E/c = h\nu/c = h/\lambda$
- Can we also talk about matter behaving both as a wave and a particle? *Yes.*
- Particles can exhibit interference effects associated with wave behavior.
- Wavelength $\lambda = h/P = h / mv$

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Wave Behavior in electrons

- Nonrelativistic approximation:
  KE = (1/2)mv^2 so λ = h/(mv) = h(2(KE)m)$^{-1/2}$
- Further, since the angular momentum mvr is quantized (mvr = nh/(2p)), we can say
  $2\pi r = n\lambda$
- So we can say that the circumference of the electron's orbit is an integer multiple of the electron's wavelength! Standing waves!

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Assignment associated with this lecture:

- Alpen, chapter 2, problem 1:
  Assume an oscillating spring that has a spring constant, k, of 20 Nm^{-1}, a mass of 1 kg, and an amplitude of 1 cm. If Planck's radiation formula describes the behavior of this system, what is the quantum number, n. What is ΔE if n changes by 1?
  The frequency of a simple oscillator is given by
  $\nu = (1/2\pi)\,(k/m)^{1/2}$

Assignment, continued:

- Alpen, ch.2, problem 4:
  A proposed surface for a photoelectric light detector has a work function of $2.0 * 10^{-19}$ J. What is the minimum frequency of radiation that it will detect? What will be the maximum kinetic energy of electrons ejected from the surface when it is irradiated with light at 3550Å?

Assignment, continued:

- Alpen, chapter 2, problem 5:
  In the previous problem, what is the de Broglie wavelength of the maximum kinetic energy electron emitted from the surface? What is its momentum?

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Assignment, concluded:

- (from my head...):
  The Advanced Photon Source (APS) at Argonne National Laboratory produces X-rays from electrons that have been accelerated to an energy of approximately 7 gigaelectron volts. This corresponds to an electron velocity very close to the speed of light. If an APS electron's speed is v, calculate $c-v$ in m/sec to two significant figures.

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Stable and Unstable Elements

- Every element has ≥1 unstable isotope, i.e. one that undergoes radioactive decay
- Most elements with $Z < 92$ have at least one stable isotope
- We'll examine radioactivity in terms of the transitions under which an atom decays
- Radioactivity has various influences on biological tissue:
  - Ionization of biological macromolecules
  - Indirect effects, often via free radicals
  - Medical applications: therapy, diagnostics, . . .

Radioactivity

- Nuclear Stability
- Mass Decrement
- Alpha Emission
- Negative Beta Emission
- Positive Beta Emission
- Electron Capture
- Spontaneous Fission

Rules

- Stable nuclei of even Z more numerous than odd Z.
- Stable nuclei of even N more numerous than odd N.
- Stable nuclei of even A more numerous than odd A.
- In general, stable nuclei of even A have even Z. Some exceptions exist, however, such as ^{2}H, ^{6}Li, ^{10}B, and ^{14}N
- Only two stable structures are known for which Z is greater than N

Examining what happens to the N/Z ratio in a typical alpha de

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Mass Energy of α - Particle

Ignoring binding energy

$2 * m_o c^2$(neutron) $\approx$ 1978 MeV

$2 * m_o c^2$(proton) $\approx$ 1976 MeV

Mass Energy = 3954 MeV

Kinetic Energy ~ 4 MeV

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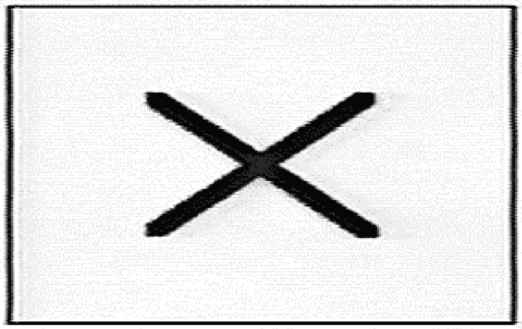
Energy - Wavelength Relationship for Photons



for the photons emitted in a positron annihilation

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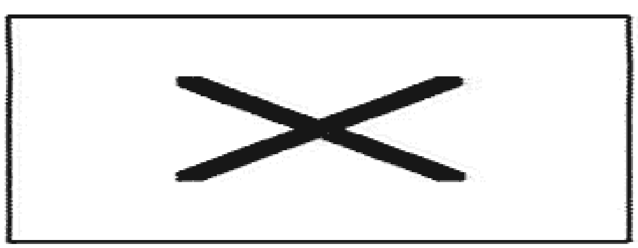
Electron Capture

Chemical Symbol

Simplified form:
Chemical Symbol

U



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Charting Decay Schemes

We can sometimes find multiple pathways, each with multiple steps, as with ^{74}As here (this is fig. 3.4, p. 37, in Alpen)

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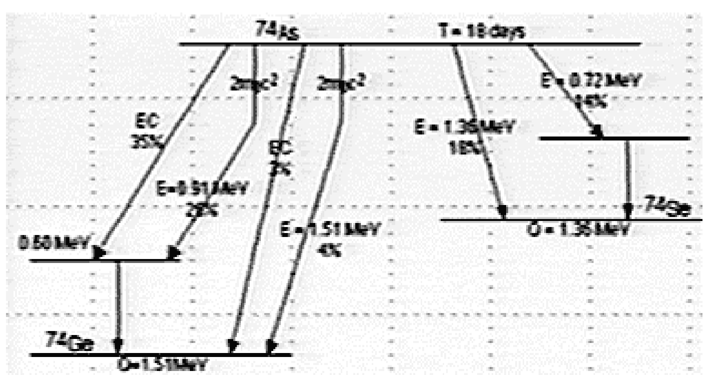

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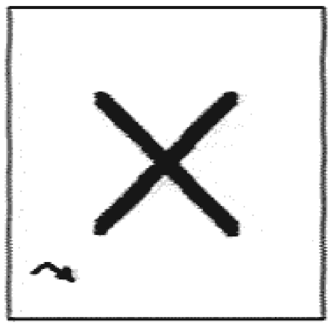
Nomenclature for Nuclei

Incorporates

- Atomic Number
- Atomic Mass Number
- Neutron Number



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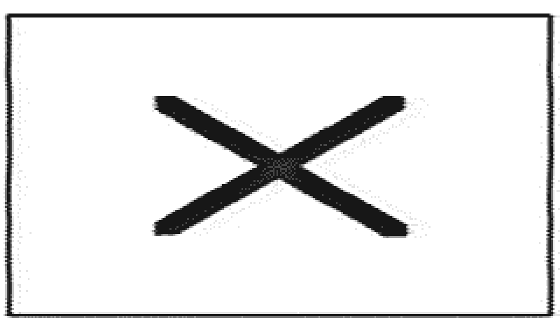
Law of Radioactivity

- Rate of disappearance is proportional to the quantity of original nuclide remaining:



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$$\frac{dN}{dt} \propto -N$$

$$\frac{dN}{dt} = -\lambda N$$

$$\frac{dN}{N} = -\lambda\, dt$$

$$\ln N = -\lambda t + K$$

$$N = e^{-\lambda t + K}$$

$$N = e^{-\lambda t} \cdot e^{K}$$

let $e^{K} = N_0 \Rightarrow N = N_0 e^{-\lambda t}$

at $t=0$, $N = N_0$

$$\frac{dN}{dt} = -\lambda N$$

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$$N = e^{-\lambda t + K}$$

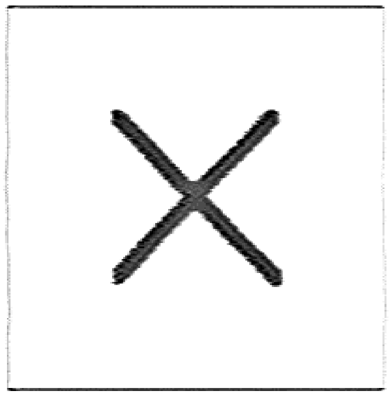
$$N = e^{-\lambda t} \cdot e^{K}$$

let $e^{K} = N_0 \Rightarrow N = N_0 e^{-\lambda t}$

at $t=0$, $N = N_0$

Half-life and activity

Half-life = time at which $N = N_0/2$



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(alf-time : $N = \frac{\cancel{N_0}}{2} = \cancel{N_0} e^{-\lambda t_{1/2}}$

$\frac{1}{2} = e^{-\lambda t_{1/2}}$, $\ln \frac{1}{2} = -\lambda t_{1/2}$

$-\ln 2 = -\lambda t_{1/2}$

$t_{1/2} = \frac{\ln 2}{\lambda}$

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Exposure: A More Detailed Definition

Exposure is defined as *charge to which the target material is exposed per unit mass*

Exposure = $\Delta Q/\Delta m$

The unit of exposure to electromagnetic radiation is the *Roentgen*

The Roentgen was defined as 1 esu/cm^3 air at STP

$\rho(air)_{STP} = 1.293 \cdot 10^{-3}$ g/cm^3

1 esu = $3.34 \cdot 10^{-10}$ Coulomb

So 1R = $3.34 \cdot 10^{-10}$ C /(1 cm^3 * $1.293 \cdot 10^{-6}$ kg/cm^3)

i.e. 1 Roentgen = $2.58 \cdot 10^{-4}$ C/kg

Assignment from Ch. 3 for Next Week

1. Chapter 3, problem 1:

The atomic mass, m, of ^{64}Cu is 63.929757 amu. It undergoes positron decay with a half-life of 12.9 h. The product of this decay is ^{64}Ni. The mass, m, of this product is 63.927956. What is the total energy of the positron and the neutrino resulting from the decay? Is the product liable to be stable or be radioactive? Why?

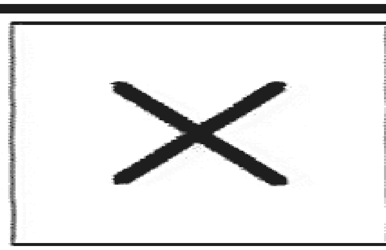
2. Chapter 3, problem 3

A source of ^{99m}Tc arrives at the laboratory for use at 10 AM on a Monday morning, at which time this daughter product is eluted for diagnostic use. The parent, ^{99}Mo, has a decay constant of 0.01039 h^{-1}. If, after the separation of the daughter, the parent was found to have an activity of $5.0 \cdot 10^9$ Bq, what is the activity of the parent and the daughter the following Thursday at 10 am?

Further Notes on Homework

- No assignment from chapter 4 until next week
- Reminder: Course web page is: http://icarus.csrri.iit.edu/radbio/
  - Includes assignments!

Interactions of Photons with Absorber



Number of photons not transmitted:

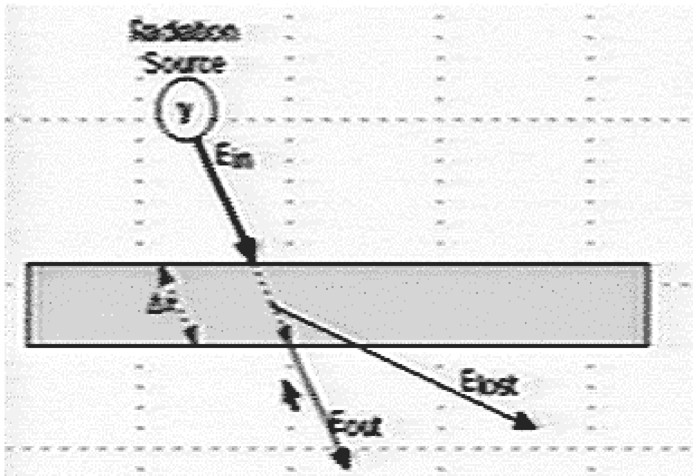
$\Delta N = \mu \Delta x N$

μ is the *linear attenuation coefficient*

Units of μ: inverse length

Energies: $E_{in} - E_{out} = E_{transferred}$

$E_{absorbed}$ **= amount that stays =** $(E_{in} - E_{out}) - E_{lost}$



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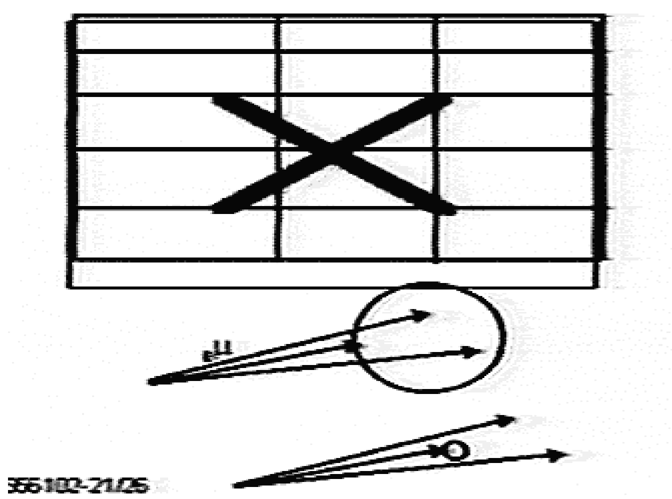
Absorption/Attenuation Coefficients



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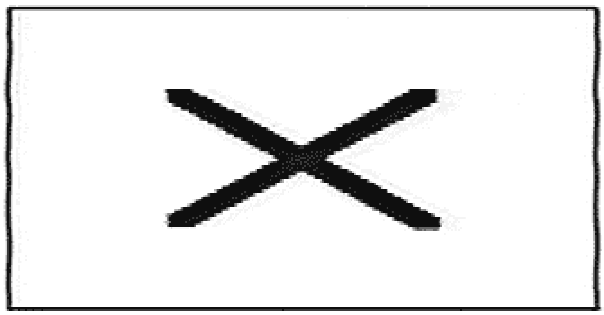
Energy Transferred and Absorbed from Photons by Carbon:

Table 4.2 in book:
Energy Transferred and Energy Absorbed for Incident Photons of Various Energies (for Carbon)



Mechanisms of Energy Transfer

Gamma Rays

- Photoelectric scattering:

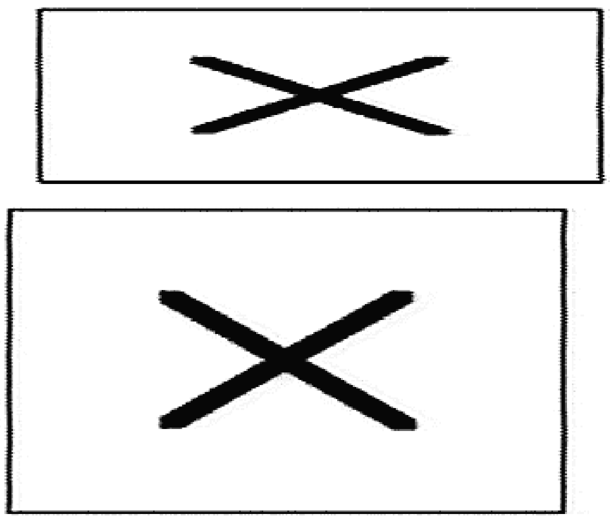
Photoelectron KE = $h\nu$ - binding energy

- Must involve bound electrons
- Partial cross section $\tau/\rho \propto (h\nu)^u$, roughly:

  u = -3 for low-Z elements

  u = -2.96 for Pb

Cross Section



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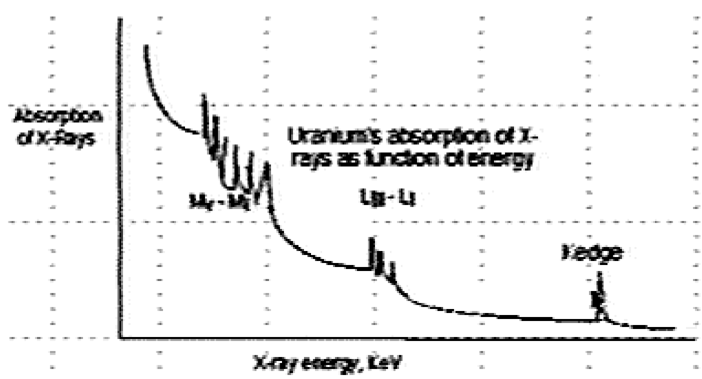

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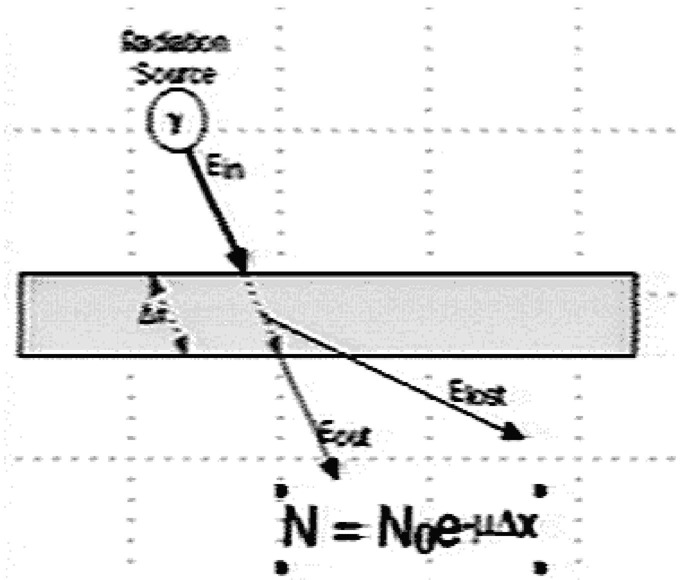

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