Today’s Outline - November 19, 2012

- Problem 5.7
- Exchange Forces
- Helium
- Multi-electron atoms

Homework Assignment #12:
Chapter 5: 6, 9, 12, 13, 32, 33
due Wednesday, November 28, 2012
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Problem 5.7

Suppose you had three particles, each of mass $m$, one in state $\psi_a(x)$, one in state $\psi_b(x)$, and one in state $\psi_c(x)$. Assuming that the three states are orthonormal, construct the three particle states representing distinguishable particles, identical bosons, and identical fermions.
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**distinguishable particles**
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distinguishable particles

This is the simplest case since we need no special symmetry
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$$\psi(x_1, x_2, x_3) = \psi_a(x_1)\psi_b(x_2)\psi_c(x_3)$$
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In this case, we need a symmetric wavefunction under exchange of any two particles. We can build this by systematically.
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$$\psi(x_1, x_2, x_3) = \left[ \psi_a(x_1)\psi_b(x_2)\psi_c(x_3) + \psi_a(x_1)\psi_c(x_2)\psi_b(x_3) \right]$$
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$$\psi(x_1, x_2, x_3) = [ \psi_a(x_1)\psi_b(x_2)\psi_c(x_3) + \psi_a(x_1)\psi_c(x_2)\psi_b(x_3) \\
+ \psi_b(x_1)\psi_c(x_2)\psi_a(x_3) + \psi_b(x_1)\psi_a(x_2)\psi_c(x_3) ]$$
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$$\psi(x_1, x_2, x_3) = \begin{bmatrix} \psi_a(x_1)\psi_b(x_2)\psi_c(x_3) + \psi_a(x_1)\psi_c(x_2)\psi_b(x_3) \\
+ \psi_b(x_1)\psi_c(x_2)\psi_a(x_3) + \psi_b(x_1)\psi_a(x_2)\psi_c(x_3) \\
+ \psi_c(x_1)\psi_a(x_2)\psi_b(x_3) + \psi_c(x_1)\psi_b(x_2)\psi_a(x_3) \end{bmatrix}$$
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In this case, we need a symmetric wavefunction under exchange of any two particles. We can build this by systematically.

\[
\psi(x_1, x_2, x_3) = \sqrt{1/6} \left[ \psi_a(x_1)\psi_b(x_2)\psi_c(x_3) + \psi_a(x_1)\psi_c(x_2)\psi_b(x_3) + \psi_b(x_1)\psi_c(x_2)\psi_a(x_3) + \psi_b(x_1)\psi_a(x_2)\psi_c(x_3) + \psi_c(x_1)\psi_a(x_2)\psi_b(x_3) + \psi_c(x_1)\psi_b(x_2)\psi_a(x_3) \right]
\]
identical fermions
identical fermions

This wavefunction needs to be antisymmetric under two particle exchange. This can be hard to do by hand but fortunately, there is a systematic way of generating these wavefunctions called **Slater determinants**.
Problem 5.7 (cont.)

identical fermions

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\[
\psi(x_1, x_2, x_3) = \sqrt{\frac{1}{6}} \left\{ \psi_a(x_1)\psi_b(x_2)\psi_c(x_3) - \psi_a(x_1)\psi_c(x_2)\psi_b(x_3) - \psi_b(x_1)\psi_a(x_2)\psi_c(x_3) + \psi_b(x_1)\psi_c(x_2)\psi_a(x_3) + \psi_c(x_1)\psi_a(x_2)\psi_b(x_3) - \psi_c(x_1)\psi_b(x_2)\psi_a(x_3) \right\}
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\[
\psi(x_1, x_2, x_3) = \begin{vmatrix}
\psi_a(x_1) & \psi_b(x_1) & \psi_c(x_1) \\
\psi_a(x_2) & \psi_b(x_2) & \psi_c(x_2) \\
\psi_a(x_3) & \psi_b(x_3) & \psi_c(x_3)
\end{vmatrix} = \sqrt{\frac{1}{6}} \left\{ \psi_a(x_1)\psi_b(x_2)\psi_c(x_3) - \psi_a(x_1)\psi_c(x_2)\psi_b(x_3) + \psi_b(x_1)\psi_a(x_2)\psi_c(x_3) - \psi_b(x_1)\psi_c(x_2)\psi_a(x_3) + \psi_c(x_1)\psi_a(x_2)\psi_b(x_3) - \psi_c(x_1)\psi_b(x_2)\psi_a(x_3) \right\}
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\[
\psi(x_1, x_2, x_3) = \det \begin{vmatrix} \psi_a(x_1) & \psi_b(x_1) & \psi_c(x_1) \\ \psi_a(x_2) & \psi_b(x_2) & \psi_c(x_2) \\ \psi_a(x_3) & \psi_b(x_3) & \psi_c(x_3) \end{vmatrix} = \sqrt{\frac{1}{6}} \{ \psi_a(x_1)[\psi_b(x_2)\psi_c(x_3) - \psi_c(x_2)\psi_b(x_3)] - \psi_b(x_1)[\psi_a(x_2)\psi_c(x_3) - \psi_c(x_2)\psi_a(x_3)] + \psi_c(x_1)[\psi_a(x_2)\psi_b(x_3) - \psi_b(x_2)\psi_a(x_3)] \}.
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\psi_a(x_3) & \psi_b(x_3) & \psi_c(x_3)
\end{vmatrix} = \sqrt{\frac{1}{6}} \left\{ \right.
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\left. \right. 
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\]

\[
= \sqrt{\frac{1}{6}} \left\{ \psi_a(x_1) \left[ \psi_b(x_2) \psi_c(x_3) \right. - \left. \psi_c(x_2) \psi_b(x_3) \right] \right\}
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This wavefunction needs to be antisymmetric under two-particle exchange. This can be hard to do by hand but fortunately, there is a systematic way of generating these wavefunctions called \textbf{Slater determinants}.

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\psi_a(x_3) & \psi_b(x_3) & \psi_c(x_3) 
\end{vmatrix}
= \sqrt{\frac{1}{6}} \left\{ \psi_a(x_1)[\psi_b(x_2)\psi_c(x_3) - \psi_c(x_2)\psi_b(x_3)] \\
- \psi_b(x_1)[\psi_a(x_2)\psi_c(x_3) - \psi_c(x_2)\psi_a(x_3)] \\
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\end{vmatrix}
\]

\[
= \sqrt{\frac{1}{6}} \left\{ \psi_a(x_1)[\psi_b(x_2)\psi_c(x_3) - \psi_c(x_2)\psi_b(x_3)] \\
- \psi_b(x_1)[\psi_a(x_2)\psi_c(x_3) - \psi_c(x_2)\psi_a(x_3)] \\
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+ \psi_c(x_1) [\psi_a(x_2) \psi_b(x_3) - \psi_b(x_2) \psi_a(x_3)] \right\}
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\psi_a(x_3) & \psi_b(x_3) & \psi_c(x_3)
\end{vmatrix}
\]

\[
= \sqrt{1/6} \{ \psi_a(x_1) [\psi_b(x_2) \psi_c(x_3) - \psi_c(x_2) \psi_b(x_3)] \\
- \psi_b(x_1) [\psi_a(x_2) \psi_c(x_3) - \psi_c(x_2) \psi_a(x_3)] \\
+ \psi_c(x_1) [\psi_a(x_2) \psi_b(x_3) - \psi_b(x_2) \psi_a(x_3)] \} \\
= \sqrt{1/6} \{ \psi_a(x_1) \psi_b(x_2) \psi_c(x_3) - \psi_a(x_1) \psi_c(x_2) \psi_b(x_3) \}
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\[ = \sqrt{\frac{1}{6}} \left\{ \psi_a(x_1)[\psi_b(x_2)\psi_c(x_3) - \psi_c(x_2)\psi_b(x_3)] \right. \\
\left. - \psi_b(x_1)[\psi_a(x_2)\psi_c(x_3) - \psi_c(x_2)\psi_a(x_3)] \\
+ \psi_c(x_1)[\psi_a(x_2)\psi_b(x_3) - \psi_b(x_2)\psi_a(x_3)] \right\} \]

\[ = \sqrt{\frac{1}{6}} \left\{ \psi_a(x_1)\psi_b(x_2)\psi_c(x_3) - \psi_a(x_1)\psi_c(x_2)\psi_b(x_3) \\
- \psi_b(x_1)\psi_a(x_2)\psi_c(x_3) + \psi_b(x_1)\psi_c(x_2)\psi_a(x_3) \\
+ \psi_c(x_1)\psi_a(x_2)\psi_b(x_3) - \psi_c(x_1)\psi_b(x_2)\psi_a(x_3) \right\} \]