A short review of modern physics

• Black-body radiation
• Photoelectric effect
• Compton scattering
• Davisson-Germer experiment
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Black body radiation

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The wavelength of the spectrum maximum \( \lambda_m \) scales inversely with temperature such that

\[
\lambda_m \propto \frac{1}{T} = \frac{2.898 \times 10^{-3}}{K^{3/2}}
\]

This proves to be a universal curve. However, the classical theoretical model (Rayleigh–Jeans) is unable to describe the low wavelength cutoff observed.

\[
\int_0^\infty u(\lambda) \, d\lambda \propto \int_0^\infty \lambda^{-4} \, d\lambda \to \infty
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![Graph showing the radiation spectrum of a black body for different temperatures. The x-axis represents wavelength in micrometers, and the y-axis represents intensity in arbitrary units. The graph shows three curves for $T=5000$, $4000$, and $3000$ K, illustrating the decrease in intensity as the wavelength increases.](image-url)
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$$\int_0^\infty u(\lambda) d\lambda \propto \int_0^\infty \lambda^{-4} d\lambda \rightarrow \infty$$
Planck’s solution

Consider a metallic cavity that forces the electric field of light waves to be zero at the inside surfaces.
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The resulting function for the energy distribution is

$$u(λ) \propto λ^{−5} e^{hc/λkT}$$

$$\lim_{λ \to 0} u(λ) = e^{hc/λkT}λ^{−5}$$

$$E_m = mhν, \quad m = 0, 1, 2, 3, \ldots$$
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which cuts off properly as \( \lambda \to 0 \).
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\[ u(\lambda) \propto \frac{\lambda^{-5}}{e^{hc/\lambda kT} - 1} \]

\[ \lim_{\lambda \to 0} u(\lambda) = \frac{e^{-hc/\lambda kT}}{\lambda^5} \]

\[ E_m = mh\nu, \quad m = 0, 1, 2, 3, \cdots \]
Photoelectric effect

In the photoelectric effect, a bare metal surface is exposed to light whose energy is absorbed, kicking out photo-electrons whose energy is measured by applying a negative potential and measuring the current of photoelectrons.
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Electron emission is found to depend on the color of the incident light rather than its intensity; for long wavelengths, no electrons are emitted.

- **550 nm**
  - Energy: 2.25 eV
  - $v_{\text{max}} = 2.96 \times 10^5 \text{ m/s}$

- **400 nm**
  - Energy: 3.10 eV
  - $v_{\text{max}} = 6.22 \times 10^5 \text{ m/s}$

- **700 nm**
  - Energy: 1.77 eV
  - No electrons
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In the photoelectric effect, a bare metal surface is exposed to light whose energy is absorbed, kicking out photo-electrons whose energy is measured by applying a negative potential and measuring the current of photoelectrons. Electron emission is found to depend on the color of the incident light rather than its intensity; for long wavelengths, no electrons are emitted. As the wavelength is reduced, electrons are emitted at a threshold wavelength.

\[
\begin{align*}
550 \text{ nm} & \quad 2.25 \text{ eV} & \quad v_{\text{max}} &= 2.96 \times 10^5 \text{ m/s} \\
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Electron emission is found to depend on the color of the incident light rather than its intensity; for long wavelengths, no electrons are emitted.

As the wavelength is reduced, electrons are emitted at a threshold wavelength.

The maximum velocity of the emitted electrons measured by the stopping potential increases.
Einstein (1905) explained this by reasoning that light must be quantized according to its frequency, thereby acting as a particle. The threshold wavelength is directly related to the work function, $\phi$

\[ \nu_{\text{thresh}} = \frac{\phi}{h} \]

The maximum electron velocity, $v_{\text{max}}$, is a function of the work function and the energy of the incident photons.

\[ \frac{1}{2}mv_{\text{max}}^2 = h\nu - \phi \]

The intensity of the light, not the energy of the photon, determines how many electrons are emitted.

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>Energy (eV)</th>
<th>Velocity ($v_{\text{max}}$)</th>
</tr>
</thead>
<tbody>
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$$\nu_{thresh} = \frac{\phi}{h}$$

$\nu_{thresh}$ is the threshold frequency, $\phi$ is the work function, and $h$ is Plank's constant.

### Example Calculations

- **400 nm (3.10 eV)**
  - $v_{max} = 6.22 \times 10^5 \text{ m/s}$
  - Maximum electron velocity

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The maximum electron velocity, $v_{\text{max}}$, is a function of the work function and the energy of the incident photons:

$$ v_{\text{max}} = \sqrt{\frac{h \nu - \phi}{m}} $$

The intensity of the light, not the energy of the photon, determines how many electrons are emitted.

- **400 nm, 3.10 eV**: $v_{\text{max}} = 2.96 \times 10^5 \text{ m/s}$
- **550 nm, 2.25 eV**: $v_{\text{max}} = 6.22 \times 10^5 \text{ m/s}$
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In 1923, Arthur Compton measured the scattering of x-rays from a carbon foil as a function of exit angle using a single crystal as an energy analyzer, to measure the x-ray spectrum in the forward scattering (zero degrees) direction, he observed a single sharp peak centered at the incident x-ray wavelength as the scattering angle was increased, x-rays at longer wavelengths (lower energies) than the incident energy were observed.
Compton scattering experiment

As the angle was further increased, the spectrum showed two peaks, one at the incident wavelength and a broader one at the longer wavelength.
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This data could be explained by treating the x-rays as particles which interact with the electrons in the carbon atoms of the foil through an inelastic collision.
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The peak wavelength of the Compton scattering peak can be calculated by applying relativistic scattering theory.
Compton scattering phenomenon

A photon-electron collision

\[ \vec{p} = \hbar \vec{k} \rightarrow \vec{p}' = 2\pi \hbar \lambda \]

Treat the electron relativistically and conserve energy and momentum

\[ mc^2 + hc\lambda = hc\lambda' + \gamma mc^2 \text{(energy)} \]

\[ \hbar \lambda = \hbar \lambda' \cos \phi + \gamma mv \cos \theta \text{(x-axis)} \]

\[ 0 = \hbar \lambda' \sin \phi + \gamma mv \sin \theta \text{(y-axis)} \]
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\[ mc^2 + \frac{hc}{\lambda} = \frac{hc}{\lambda'} + \gamma mc^2 \quad \text{(energy)} \]

\[ \frac{\hbar}{\lambda} = \frac{\hbar}{\lambda'} \cos \varphi + \gamma mv \cos \theta \quad \text{(x-axis)} \]
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Treat the electron relativistically and conserve energy and momentum

\[ mc^2 + \frac{hc}{\lambda} = \frac{hc}{\lambda'} + \gamma mc^2 \quad \text{(energy)} \]

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Compton scattering derivation

squaring the momentum equations
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\[
\left( \frac{h}{\lambda} - \frac{h}{\lambda'} \cos \varphi \right)^2 = \gamma^2 m^2 v^2 \cos^2 \theta
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Compton scattering derivation

squaring the momentum equations

\[
\left( \frac{h}{\lambda} - \frac{h}{\lambda'} \cos \varphi \right)^2 = \gamma^2 m^2 v^2 \cos^2 \theta \\
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now add them together,

\[ \gamma^2 m^2 v^2 \left( \sin^2 \theta + \cos^2 \theta \right) = \left( \frac{h}{\lambda} - \frac{h}{\lambda'} \cos \varphi \right)^2 + \left( -\frac{h}{\lambda'} \sin \varphi \right)^2 \]
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\]

now add them together, substitute \( \sin^2 \theta + \cos^2 \theta = 1 \), expand the squares,

\[
\gamma^2 m^2 v^2 \left( \sin^2 \theta + \cos^2 \theta \right) = \left( \frac{h}{\lambda} - \frac{h}{\lambda'} \cos \varphi \right)^2 + \left( -\frac{h}{\lambda'} \sin \varphi \right)^2
\]
\[
\gamma^2 m^2 v^2 = \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda \lambda'} \cos \varphi + \frac{h^2}{\lambda'^2} \sin^2 \varphi + \frac{h^2}{\lambda'^2} \cos^2 \varphi
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\[
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m^2 v^2 = \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda\lambda'} \cos \varphi + \frac{h^2}{\lambda'^2} \\
\frac{1}{1-\beta^2} = \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda\lambda'} \cos \varphi + \frac{h^2}{\lambda'^2}
\]
Compton scattering derivation

squaring the momentum equations

\[
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now add them together, substitute \( \sin^2 \theta + \cos^2 \theta = 1 \), expand the squares, and \( \sin^2 \varphi + \cos^2 \varphi = 1 \), then rearrange and substitute \( v = \beta c \)

\[
\gamma^2 m^2 v^2 (\sin^2 \theta + \cos^2 \theta) = \left( \frac{h}{\lambda} - \frac{h}{\lambda'} \cos \varphi \right)^2 + \left( -\frac{h}{\lambda'} \sin \varphi \right)^2
\]

\[
\gamma^2 m^2 v^2 = \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda\lambda'} \cos \varphi + \frac{h^2}{\lambda'^2} \sin^2 \varphi + \frac{h^2}{\lambda'^2} \cos^2 \varphi
\]

\[
\frac{m^2 c^2 \beta^2}{1 - \beta^2} = \frac{m^2 v^2}{1 - \beta^2} = \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda\lambda'} \cos \varphi + \frac{h^2}{\lambda'^2}
\]
Compton scattering derivation (cont.)

Now take the energy equation and square it,

\[
\left( mc^2 + \frac{hc}{\lambda} - \frac{hc}{\lambda'} \right)^2 = \gamma^2 m^2 c^4 = \frac{m^2 c^4}{1 - \beta^2}
\]
Now take the energy equation and square it, then solve it for $\beta^2$

\[
\left( mc^2 + \frac{hc}{\lambda} - \frac{hc}{\lambda'} \right)^2 = \gamma^2 m^2 c^4 = \frac{m^2 c^4}{1 - \beta^2}
\]

\[
\beta^2 = 1 - \frac{m^2 c^4}{\left( mc^2 + \frac{hc}{\lambda} - \frac{hc}{\lambda'} \right)^2}
\]
Compton scattering derivation (cont.)

Now take the energy equation and square it, then solve it for $\beta^2$ which is substituted into the equation from the momentum.

$$\left(mc^2 + \frac{hc}{\lambda} - \frac{hc}{\lambda'}\right)^2 = \gamma^2 m^2 c^4 = \frac{m^2 c^4}{1 - \beta^2}$$

$$\beta^2 = 1 - \frac{m^2 c^4}{\left(mc^2 + \frac{hc}{\lambda} - \frac{hc}{\lambda'}\right)^2}$$

$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - 2\frac{h^2}{\lambda \lambda'} \cos \varphi = \frac{m^2 c^2 \beta^2}{1 - \beta^2}$$
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\beta^2 = 1 - \frac{m^2 c^4}{(mc^2 + \frac{hc}{\lambda} - \frac{hc}{\lambda'})^2}
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\[
\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda \lambda'} \cos \varphi = \frac{m^2 c^2 \beta^2}{1 - \beta^2}
\]

\[
= \frac{1}{c^2} \left(mc^2 + \frac{hc}{\lambda} - \frac{hc}{\lambda'}\right)^2 - m^2 c^2
\]
After expansion, cancellation, and rearrangement, we obtain:

\[
\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - 2\frac{h^2}{\lambda\lambda'} \cos \varphi = \left( mc + \frac{h}{\lambda} - \frac{h}{\lambda'} \right)^2 - m^2 c^2
\]
Compton scattering derivation (cont.)

After expansion,

\[
\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda \lambda'} \cos \varphi = \left( mc + \frac{h}{\lambda} - \frac{h}{\lambda'} \right)^2 - m^2 c^2
\]

\[= m^2 c^2 + \frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} + \frac{2mch}{\lambda} - \frac{2mch}{\lambda'} + \frac{2h^2}{\lambda \lambda'} - m^2 c^2 \]
Compton scattering derivation (cont.)

After expansion, cancellation, and rearrangement, we obtain

\[
\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda' r^2} - \frac{2h^2}{\lambda \lambda'} \cos \varphi = \left( mc + \frac{h}{\lambda} - \frac{h}{\lambda'} \right)^2 - m^2 c^2
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The Davisson-Germer paper was published in the Proceedings of the National Academy of Sciences in 1928 and an historical account of the discovery was published in 1978 in Physics Today.
The Schrödinger equation
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- The 1-D Schrödinger equation
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1D Schrödinger equation

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matrix mechanics was worked out in parallel by Heisenberg, Born, and Pascual and relativistic approaches were developed by Dirac and Pauli
Deriving the Schrödinger equation

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carlossegre | PHYS 405 - Fundamentals of Quantum Theory I
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Inspired by wave optics, Schrödinger started with the wave equation for electromagnetic radiation

\[ E(x,t) = E_0 e^{i(kx - \omega t)} \]

Taking the derivatives and substituting results in the dispersion relation for photons

\[ 0 = \frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} \]

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The dispersion relation for a non-relativistic particle must be consistent with the classical energy

\[ E = \frac{p^2}{2m} = \hbar \omega \]

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Therefore, we need a wave equation which gives this dispersion relation when applied to a traveling matter plane wave, \( \Psi(x,t) = \psi_0 e^{i(kx - \omega t)} \)

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