Today’s Outline - January 28, 2013

- First order PT examples
- Second order PT
- Degenerate PT

Reading Assignment: Chapter 6.3

Homework Assignment #02:
Chapter 5: 27, 30; Chapter 6: 1, 4, 6, 29
due Monday, February 4, 2013
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Unperturbed Hamiltonian, $H^0$
with solutions $\psi_n^0$
First order perturbation theory review

Unperturbed Hamiltonian, $H^0$
with solutions $\psi^0_n$

\[ H^0 \psi^0_n = E^0_n \psi^0_n \]
First order perturbation theory review

Unperturbed Hamiltonian, $H^0$
with solutions $\psi_n^0$

$$H^0 \psi_n^0 = E_n^0 \psi_n^0$$
$$\langle \psi_n^0 | \psi_m^0 \rangle = \delta_{nm}$$
First order perturbation theory review

Unperturbed Hamiltonian, $H^0$
with solutions $\psi_n^0$

"perturbed" Hamiltonian,

$$H = H^0 + H'$$

$$H^0 \psi_n^0 = E_n^0 \psi_n^0$$

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\[ H \psi_n = E_n \psi_n \]
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“perturbed” Hamiltonian,

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$$H \psi_n = E_n \psi_n$$

$$\psi_n = \sum_m c_m(n) \psi_m^0$$
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Unperturbed Hamiltonian, $H^0$
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"perturbed" Hamiltonian,

$$H = H^0 + H'$$

first order energy correction

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$$E_n^1 = \langle \psi_n^0 | H' | \psi_n^0 \rangle$$
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“perturbed” Hamiltonian,

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H = H^0 + H'
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first order energy correction

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first order wavefunction correction

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first order wavefunction correction

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\langle \psi_n^0 | \psi_m^0 \rangle = \delta_{nm}
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H \psi_n = E_n \psi_n
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\psi_n = \sum_m c_m^{(n)} \psi_m^0
\]

\[
E_n^1 = \langle \psi_n^0 | H' | \psi_n^0 \rangle
\]

\[
\psi_n^1 = \sum_{m \neq n} \frac{\langle \psi_m^0 | H' | \psi_n^0 \rangle \psi_m^0}{(E_n^0 - E_m^0)}
\]
Example 6.1

Constant potential, $V_0$ in an infinite square well from $x = 0$ to $x = a$
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The unperturbed wavefunctions
Example 6.1

Constant potential, $V_0$ in an infinite square well from $x = 0$ to $x = a$

The unperturbed wavefunctions

$$\psi_n^0(x) = \sqrt{\frac{2}{a}} \sin \left( \frac{n\pi}{a} x \right)$$
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$$\psi^0_n(x) = \sqrt{\frac{2}{a}} \sin \left( \frac{n\pi}{a} x \right)$$

the perturbing potential is $H' = V_0$
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$$\psi_0^0(x) = \sqrt{\frac{2}{a}} \sin \left( \frac{n\pi}{a} x \right)$$

the perturbing potential is $H' = V_0$ and the first order energy correction
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The unperturbed wavefunctions

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\psi_n^0(x) = \sqrt{\frac{2}{a}} \sin \left( \frac{n\pi}{a} x \right)
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$$
E_n^1 = \langle \psi_n^0 | V_0 | \psi_n^0 \rangle
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Constant potential, $V_0$ in an infinite square well from $x = 0$ to $x = a$

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$$\psi_n^0(x) = \sqrt{\frac{2}{a}} \sin \left( \frac{n\pi}{a} x \right)$$

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$$E_n^1 = \langle \psi_n^0 | V_0 | \psi_n^0 \rangle = V_0 \langle \psi_n^0 | \psi_n^0 \rangle$$
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$$\psi_n^0(x) = \sqrt{\frac{2}{a}} \sin \left( \frac{n\pi}{a} x \right)$$

the perturbing potential is $H' = V_0$ and the first order energy correction

this is an exact solution with

$$E_n^1 = \langle \psi_n^0 | V_0 | \psi_n^0 \rangle = V_0 \langle \psi_n^0 | \psi_n^0 \rangle = V_0$$

$$\psi_n \equiv \psi_n^0$$
Example 6.1

Constant potential, \( V_0 \) in an infinite square well from \( x = 0 \) to \( x = a \)

The unperturbed wavefunctions

\[ \psi_0^0(x) = \sqrt{\frac{2}{a}} \sin \left( \frac{n\pi}{a} x \right) \]

the perturbing potential is \( H' = V_0 \) and the first order energy correction

\[ E_1^n = \langle \psi_0^n | V_0 | \psi_0^n \rangle = V_0 \langle \psi_0^n | \psi_0^n \rangle = V_0 \]

this is an exact solution with \( \psi_n \equiv \psi_0^n \)

suppose the perturbing potential extends only from \( 0 < x < a/2 \)?
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Constant potential, $V_0$ in an infinite square well from $x = 0$ to $x = a$

The unperturbed wavefunctions

$$\psi_n^0(x) = \sqrt{\frac{2}{a}} \sin \left(\frac{n\pi}{a} x\right)$$

the perturbing potential is $H' = V_0$ and the first order energy correction

$$E_n^1 = \langle \psi_n^0 | V_0 | \psi_n^0 \rangle = V_0 \langle \psi_n^0 | \psi_n^0 \rangle = V_0$$

this is an exact solution with

$$\psi_n \equiv \psi_n^0$$

suppose the perturbing potential extends only from $0 < x < a/2$?

$$E_n^1 = \frac{2V_0}{a} \int_0^{a/2} \sin^2 \left(\frac{n\pi}{a} x\right) dx$$
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Constant potential, \( V_0 \) in an infinite square well from \( x = 0 \) to \( x = a \)

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\psi_n^0(x) = \sqrt{\frac{2}{a}} \sin \left( \frac{n\pi}{a} x \right)
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E_n^1 = \langle \psi_n^0 | V_0 | \psi_n^0 \rangle = V_0 \langle \psi_n^0 | \psi_n^0 \rangle = V_0
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this is an exact solution with

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\psi_n \equiv \psi_n^0
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suppose the perturbing potential extends only from \( 0 < x < a/2 \)?

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E_n^1 = \frac{2V_0}{a} \int_0^{a/2} \sin^2 \left( \frac{n\pi}{a} x \right) dx = \frac{2V_0}{a} \int_0^{a/2} \frac{1}{2} \left[ 1 - \cos \left( \frac{2n\pi}{a} x \right) \right] dx
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$$= \frac{V_0}{a} \left[ x - \frac{a}{2n\pi} \sin \left(\frac{2n\pi}{a} x\right) \right]_0^{a/2}$$
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suppose the perturbing potential extends only from $0 < x < a/2$?

$$E_n^1 = \frac{2V_0}{a} \int_0^{a/2} \sin^2 \left( \frac{n\pi}{a} x \right) \, dx = \frac{2V_0}{a} \int_0^{a/2} \frac{1}{2} \left[ 1 - \cos \left( \frac{2n\pi}{a} x \right) \right] \, dx$$

$$= \frac{V_0}{a} \left[ x - \frac{a}{2n\pi} \sin \left( \frac{2n\pi}{a} x \right) \right]_0^{a/2} = \frac{V_0}{a} \left[ \frac{a}{2} + 0 \right]$$
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$$E_n^1 = \langle \psi_n^0 | V_0 | \psi_n^0 \rangle = V_0 \langle \psi_n^0 | \psi_n^0 \rangle = V_0$$

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suppose the perturbing potential extends only from $0 < x < a/2$?

$$E_n^1 = \frac{2V_0}{a} \int_0^{a/2} \sin^2\left(\frac{n\pi}{a}x\right) dx = \frac{2V_0}{a} \int_0^{a/2} \frac{1}{2} \left[ 1 - \cos\left(\frac{2n\pi}{a}x\right) \right] dx$$

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$$= \frac{V_0}{a} \left[ x - \frac{a}{2n\pi} \sin \left( \frac{2n\pi}{a} x \right) \right]_0^{a/2} = \frac{V_0}{a} \left[ \frac{a}{2} + 0 \right] = \frac{V_0}{2}$$

this is not an exact solution but the first term in a series of energy correction terms