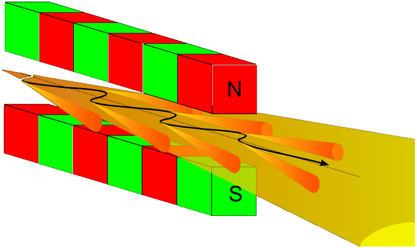
Wigglers and Undulators

Wiggler



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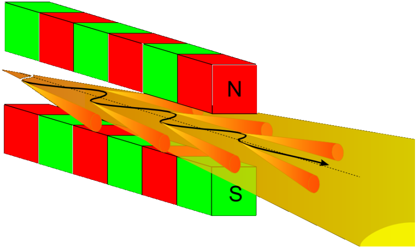

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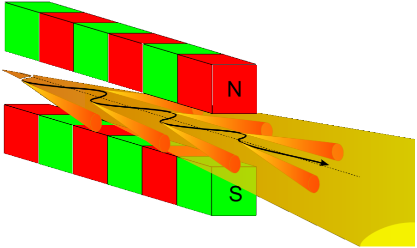

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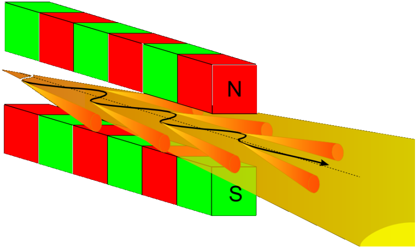

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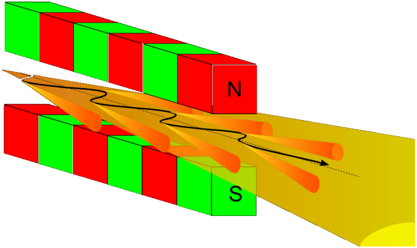

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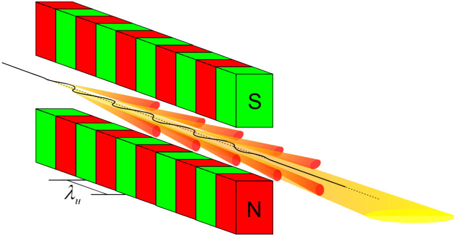

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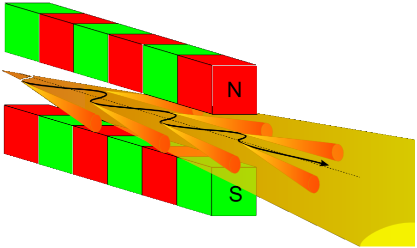

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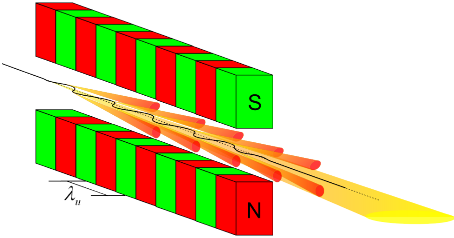

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Wigglers and Undulators

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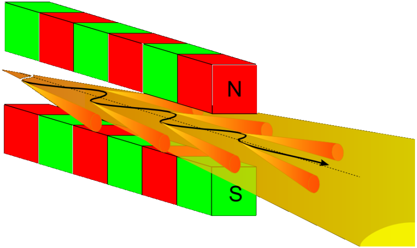

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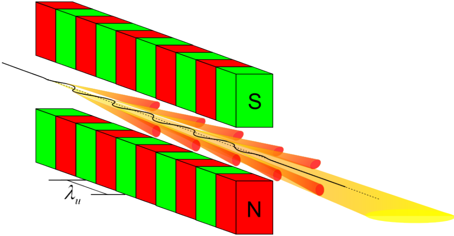

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Wigglers and Undulators

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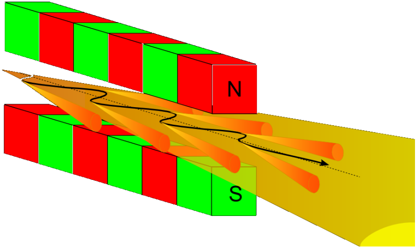

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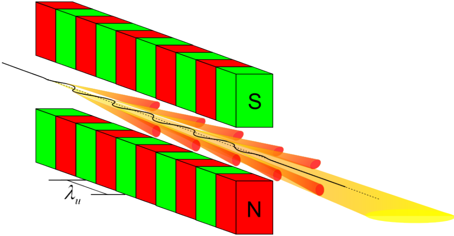

Different from bending magnet:

- shallow bends $\rightarrow$ small source
- interference effects $\rightarrow$ highly peaked spectrum

Wiggler Radiation

- The electron's trajectory through a wiggler can be considered as a series of short circular arcs, each like a bending magnet

$$Power[\text{kW}] = 1.266\mathcal{E}_e^2[\text{GeV}]B[\text{T}]L[\text{m}]I[\text{A}]$$

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- This results in a significantly higher power load on all downstream components

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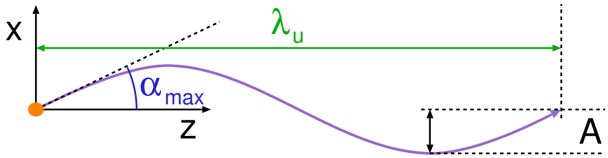
Undulator Characterization



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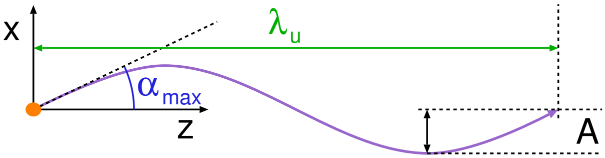

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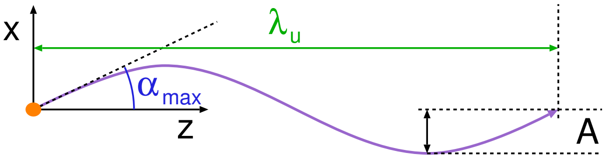

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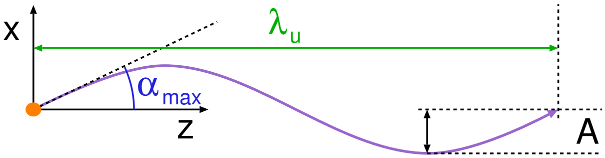

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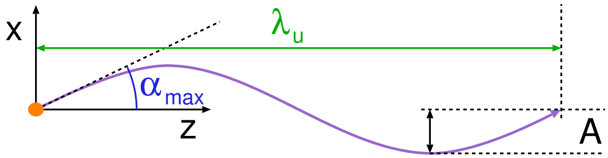

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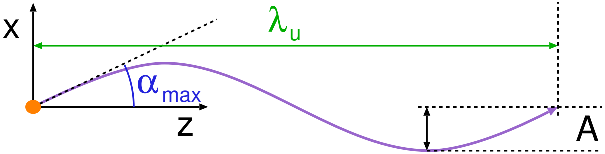

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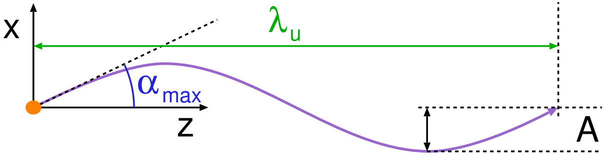

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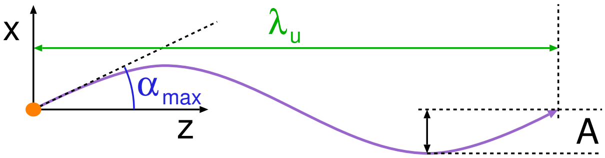

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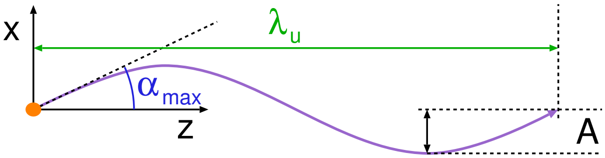

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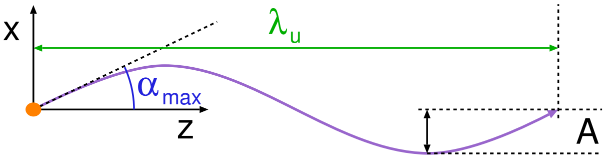

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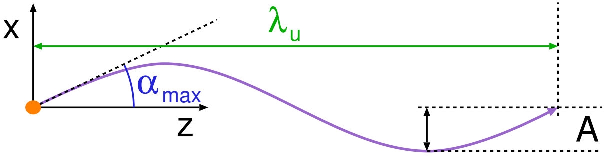

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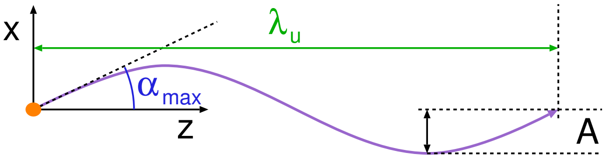

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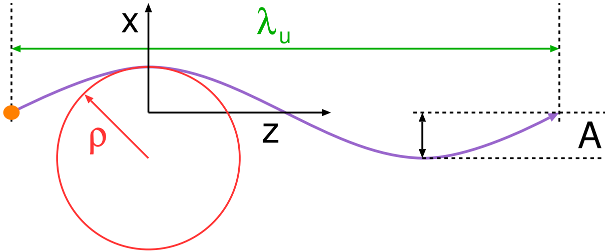
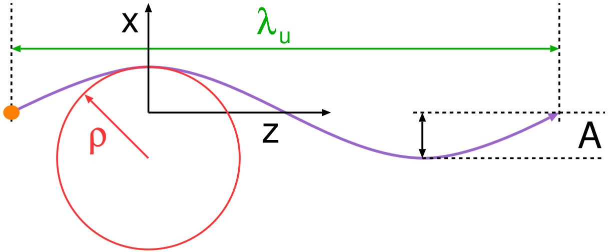
Circular Path Approximation



Consider the trajectory of the electron along one period of the undulator.

Circular Path Approximation



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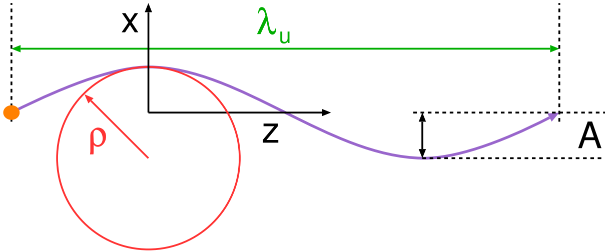

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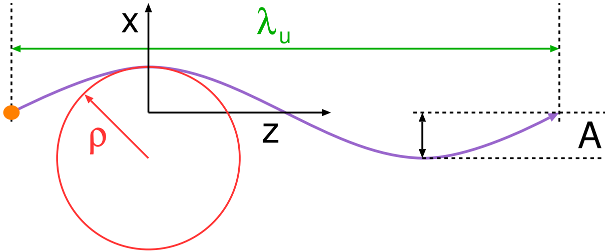

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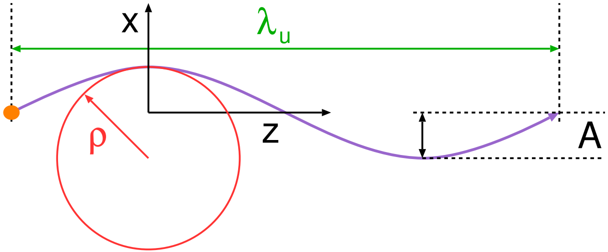

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Radius of Curvature

From the equation for a circle:

$$x = A - \rho + \sqrt{\rho^2 - z^2}$$

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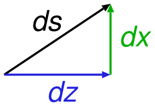
Electron Path Length

The displacement ds of the electron can be expressed in terms of the two coordinates, x and z as:



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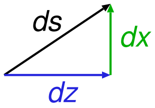

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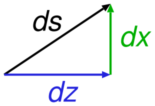

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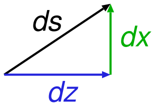

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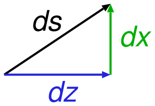

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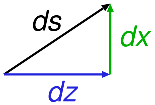

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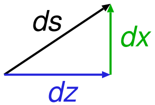
Now calculate the length of the path traveled by the electron over one period of the undulator

Electron Path Length

The displacement ds of the electron can be expressed in terms of the two coordinates, x and z as:



$$\begin{aligned} ds &= \sqrt{(dx)^2 + (dz)^2} \\ &= \sqrt{1 + \left(\frac{dx}{dz}\right)^2} dz \end{aligned}$$

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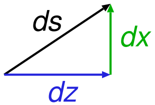

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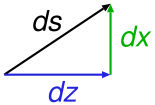

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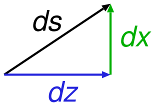

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$$K = 0.934 \cdot 3.3[\text{cm}] \cdot 0.6[\text{T}]$$

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For APS Undulator A, $\lambda_u = 3.3\text{cm}$ and $B_o = 0.6\text{T}$ at closed gap, so

$$K = 0.934 \cdot 3.3[\text{cm}] \cdot 0.6[\text{T}] = 1.85$$