Wigglers and Undulators

Wiggler

Just like bending magnet except:
• larger \( \vec{B} \rightarrow E \)
• higher
• more bends \( \rightarrow \) power

Undulator
Different from bending magnet:
• shallow bends \( \rightarrow \) small source
• interference effects \( \rightarrow \) highly peaked spectrum
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Wiggler Radiation

- The electron’s trajectory through a wiggler can be considered as a series of short circular arcs, each like a bending magnet.

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- The magnetic field varies along the length of the wiggler and is higher than that in a bending magnet, having an average value of $B_{rms} = B_o/\sqrt{2}$.

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- The magnetic field varies along the length of the wiggler and is higher than that in a bending magnet, having an average value of $B_{\text{rms}} = B_0/\sqrt{2}$.
- This results in a significantly higher power load on all downstream components.

$$\text{Power}[\text{kW}] = 0.633 \epsilon_e^2 [\text{GeV}] B_0^2 [\text{T}] L [\text{m}] I [\text{A}]$$
Undulator radiation is characterized by three parameters:

- The energy of the electrons, $\gamma$
- The wavelength, $\lambda_u$, of its magnetic field
- The maximum angular deviation of the electron, $\alpha_{\text{max}}$

We can, therefore write:

$$x = A \sin (k \lambda_u z)$$

$$\alpha_{\text{max}} = \left| \frac{dx}{dz} \right|_{z=0} = A k \lambda_u \cos (k \lambda_u z)$$

Define a dimensionless quantity, $K$, which scales $\alpha_{\text{max}}$ to the natural opening angle of the radiation, $1/\gamma$:

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We can, therefore write:

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$$K = \alpha_{max} \gamma$$
Consider the trajectory of the electron along one period of the undulator.
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The equation of the circle which approximates the arc is: 

$$\rho^2 = (x + (\rho - A))^2 + z^2$$
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Circular Path Approximation

Consider the trajectory of the electron along one period of the undulator. Since the curvature is small, the path can be approximated by an arc or a circle of radius $\rho$ whose origin lies at $x = -(\rho - A)$ and $z = 0$. The equation of the circle which approximates the arc is:

$$\rho^2 = [x + (\rho - A)]^2 + z^2$$

$$x + (\rho - A) = \sqrt{\rho^2 - z^2}$$
Radius of Curvature

From the equation for a circle:

\[ x = A - \rho + \sqrt{\rho^2 - z^2} \]
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\[ \approx A - \rho + \rho \left(1 - \frac{1}{2} \frac{z^2}{\rho^2}\right) \]
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For the undulating path:

\[ x = A \cos(kuz) \]

\[ \approx A \left(1 - \frac{1}{2} k^2 u^2 z^2 \right) \]

\[ \approx A - \frac{A k^2 u z^2}{2} \]
Radius of Curvature

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For the undulating path:

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Combining, we have

\[ \frac{1}{\rho} = Ak_u^2 \]

For the undulating path:

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Combining, we have

\[ \frac{1}{\rho} = Ak_u^2 \quad \rightarrow \quad \rho = \frac{1}{Ak_u^2} \]

For the undulating path:

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Electron Path Length

The displacement \( ds \) of the electron can be expressed in terms of the two coordinates, \( x \) and \( z \) as:

\[
ds = \sqrt{(dx)^2 + (dz)^2}
\]

Now calculate the length of the path traveled by the electron over one period of the undulator:

\[
S = \int_0^\lambda u \sqrt{1 + (dx/dz)^2} \, dz
\]

\[
\approx \int_0^\lambda u \left(1 + \frac{1}{2} A^2 k^4 u z^2 \right) \, dz
\]

\[
= \left[z + \frac{1}{6} A^2 k^4 u z^3\right]_0^\lambda u
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The displacement $ds$ of the electron can be expressed in terms of the two coordinates, $x$ and $z$ as:

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$$dx \frac{dz}{dz} = \frac{d}{dz} \left( A - \frac{Ak_u^2z^2}{2} \right)$$
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The displacement $ds$ of the electron can be expressed in terms of the two coordinates, $x$ and $z$ as:

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Now calculate the length of the path traveled by the electron over one period of the undulator

$$S\lambda_u = \int_{0}^{\lambda_u} \sqrt{1 + \left(\frac{dx}{dz}\right)^2} \, dz$$
Electron Path Length

The displacement $ds$ of the electron can be expressed in terms of the two coordinates, $x$ and $z$ as:

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Now calculate the length of the path traveled by the electron over one period of the undulator

$$S_{\lambda u} = \int_0^{\lambda u} \sqrt{1 + \left(\frac{dx}{dz}\right)^2} \, dz = \int_0^{\lambda u} \sqrt{1 + A^2 k_u^4 z^2} \, dz$$

$$\frac{dx}{dz} = \frac{d}{dz} \left( A - \frac{A k_u^2 z^2}{2} \right) = -A k_u^2 z$$
Electron Path Length

The displacement $ds$ of the electron can be expressed in terms of the two coordinates, $x$ and $z$ as:

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$$\approx \int_0^{\lambda u} \left( 1 + \frac{1}{2} A^2 k_u^4 z^2 \right) \, dz$$
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$$\approx \int_0^{\lambda u} \left(1 + \frac{1}{2} A^2 k_u^4 z^2\right) \, dz = \left[z + \frac{1}{6} A^2 k_u^4 z^3\right]_0^{\lambda u}$$
Electron Path Length

\[ S \lambda_u \approx \left[ z + \frac{1}{6} A^2 k_u^4 z^3 \right]_0^\lambda_u \]
Electron Path Length

\[ S \lambda_u \approx \left[ z + \frac{1}{6} A^2 k_u^4 z^3 \right] \bigg|_0^{\lambda_u} \]

\[ \approx \left( \lambda_u + \frac{1}{6} A^2 k_u^4 \lambda_u^3 \right) \]
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$S\lambda_u \approx \left[ z + \frac{1}{6} A^2 k_u^4 z^3 \right]_0^{\lambda_u}$

$\approx \left( \lambda_u + \frac{1}{6} A^2 k_u^4 \lambda_u^3 \right)$

$\approx \lambda_u \left( 1 + \frac{1}{6} A^2 k_u^4 \lambda_u^2 \right)$
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Electron Path Length

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The textbook presents a different constant factor for the second term and we will proceed using that factor for simplicity.
Electron Path Length

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The textbook presents a different constant factor for the second term and we will proceed using that factor for simplicity.

\[ S\lambda_u \approx \left( 1 + \frac{1}{4} A^2 k_u^2 \right) \]
Electron Path Length

\[ S\lambda_u \approx \left[ z + \frac{1}{6}A^2k_u^4z^3 \right]_0 \]

\approx \left( \lambda_u + \frac{1}{6}A^2k_u^4\lambda_u^3 \right)

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The textbook presents a different constant factor for the second term and we will proceed using that factor for simplicity

\[ S\lambda_u \approx \left( 1 + \frac{1}{4}A^2k_u^2 \right) \]

Using the definition for the undulator parameter \( K = \gamma A k_u \), we have
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\[ S\lambda_u \approx \left( 1 + \frac{1}{4} A^2 k_u^2 \right) \]

Using the definition for the undulator parameter \( K = \gamma Ak_u \), we have

\[ S\lambda_u \approx \left( 1 + \frac{1}{4} K^2 \right) \]
The $K$ Parameter

Given the definition $K = \gamma A k_u$, we can rewrite the radius of curvature of the electron's path in the undulator as

$$\rho \approx \gamma K k u \approx \gamma \frac{mc}{eB_o} \approx \gamma K k u eB_o.$$
The $K$ Parameter

Given the definition $K = \gamma A k_u$, we can rewrite the radius of curvature of the electron’s path in the undulator as

$$\rho = \frac{1}{Ak_u^2}$$
The $K$ Parameter

Given the definition $K = \gamma A k_u$, we can rewrite the radius of curvature of the electron’s path in the undulator as

$$\rho = \frac{1}{Ak_u^2} \quad \rightarrow \quad \rho = \frac{\gamma}{Kk_u}$$
Given the definition $K = \gamma Ak_u$, we can rewrite the radius of curvature of the electron’s path in the undulator as

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Recalling that the radius of curvature is related to the electron momentum by the Lorentz force, we have

For APS Undulator A, $\lambda u = 3.3\text{cm}$ and $B_o = 0.6\text{T}$ at closed gap, so $K = 0.934 \cdot 3.3\text{cm} \cdot 0.6\text{T} = 1.85$. 

C. Segre (IIT)
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