How to Make a Fresnel Lens

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aspect ratio too large for a stable structure and absorption would be too large!
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This material can be removed and the remaining material collapsed to produce a Fresnel lens which has the same optical properties as the parabolic lens as long as $f \gg N\Lambda$ where $N$ is the number of zones.
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\nu = \frac{h(x)}{\Lambda} \quad \xi = \frac{x}{\sqrt{2\lambda_0 f}}
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Since \( \nu = \xi^2 \), the position of the \( N^{th} \) zone is \( \xi_N = \sqrt{N} \) and the scaled width of the \( N^{th} \) (outermost) zone is
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The diameter of the entire lens is thus

\[ 2\Delta \xi_N = 2\sqrt{N} \approx \frac{1}{2} \Delta \xi_N N \]
Fresnel Lens Dimensions

\[ \Delta \xi_N = \xi_N - \xi_{N-1} = \sqrt{N} - \sqrt{N - 1} \]

\[ = \sqrt{N} \left( 1 - \sqrt{1 - \frac{1}{N}} \right) \]

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In terms of the unscaled variables

$$\Delta x_N = \Delta \xi_N \sqrt{2\lambda_o f}$$
Fresnel Lens Example

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If we take

\[ \lambda_o = 1\text{Å} = 1 \times 10^{-10}\text{m} \]
\[ f = 50\text{cm} = 0.5\text{m} \]
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\Delta x_N = 5 \times 10^{-7}\text{m} \quad d_N = 2 \times 10^{-4}\text{m} = 100\mu\text{m}
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In practice, since the outermost zones are very small, zone plates are generally fabricated as alternating zones (rings for 2D) of materials with a large Z-contrast, such as Au/Si or W/C.
Making a Fresnel Zone Plate

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In practice, since the outermost zones are very small, zone plates are generally fabricated as alternating zones (rings for 2D) of materials with a large Z-contrast, such as Au/Si or W/C.

This kind of zone plate is not as efficient as a true Fresnel lens would be in the x-ray regime. Nevertheless, efficiencies up to 35% have been achieved.
The shape of an ideal mirror is an ellipse, where any ray coming from one focus will be projected to the second focus.
Tangential Focusing Mirror

The shape of an ideal mirror is an ellipse, where any ray coming from one focus will be projected to the second focus. Consider a 1:1 focusing mirror. For an ellipse the sum of the distances from any point on the ellipse to the foci is a constant.

\[ F_1 P + F_2 P = 2a \]
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\[ \sin \theta = \frac{b}{a} = \frac{b}{2f} \]
Sagittal Focusing Mirror

Ellipses are hard figures to make, so usually, they are approximated by circles. In the case of sagittal focusing, an ellipsoid of revolution with diameter $2b$, is used for focusing.

\[
\rho_{\text{sagittal}} = b = 2f \sin \theta
\]

The tangential focus is also usually approximated by a circular cross-section with radius \[
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