Term: Spring 2018
Meetings: Tuesday & Thursday 13:50-15:05
Location: 213 Stuart Building

Instructor: Carlo Segre
Office: 136A Life Sciences
Phone: 312.567.3498
e-mail: segre@iit.edu


Web Site: http://csrri.iit.edu/~segre/phys570/18S
Course objectives

- Understand the means of production of synchrotron x-ray radiation
Course objectives

• Understand the means of production of synchrotron x-ray radiation

• Understand the function of various components of a synchrotron beamline
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• Be able to make an oral presentation of a synchrotron radiation research topic
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• Be able to write a General User Proposal in the format used by the Advanced Photon Source
Course syllabus

• Focus on applications of synchrotron radiation
Course syllabus

- Focus on applications of synchrotron radiation
- Homework assignments

- In-class student presentations on research topics
- Choose a research article which features a synchrotron technique
- Timetable will be posted
- Final project - writing a General User Proposal
- Start thinking about a suitable project right away
- Make proposal and get approval before starting
- Visits to Advanced Photon Source (outside class, not required)
  - All students who plan to attend will need to request badges from APS
  - Use MRCAT (Sector 10) as location of experiment
  - Use Carlo Segre as local contact
  - State that your beamtime will be in the second week of March
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Grading scale

A  –  80%  to  100%
B  –  65%  to  80%
C  –  50%  to  65%
E  –  0%  to  50%
Topics to be covered (at a minimum)

• X-rays and their interaction with matter
• Sources of x-rays
• Refraction and reflection from interfaces
• Kinematical diffraction
• Diffraction by perfect crystals
• Small angle scattering
• Photoelectric absorption
• Resonant scattering
• Imaging
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Today's outline - January 09, 2018

- The big picture
- History of x-ray sources
- X-ray interactions with matter
- Thomson scattering
- Atomic form factor

Reading Assignment: Chapter 1.1–1.6; 2.1–2.2
Today’s outline - January 09, 2018

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Why synchrotron radiation?

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The broad range of techniques make synchrotron x-ray sources to nearly any science or engineering field
A bit about my research...
History of x-ray sources

- 1895 x-rays discovered by William Röntgen

• 1st generation synchrotrons initially used in parasitic mode (SSRL, CHESS)
• 2nd generation were dedicated sources (NSLS, SRC, CAMD)
• 3rd generation featured insertion devices (APS, ESRF, ALS)
• 4th generation are free electron lasers (LCLS, XFEL)
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\[ E(r, t) = \hat{e}E_0 e^{i(k \cdot r - \omega t)} \]

where \( \hat{e} \) is a unit vector in the direction of the electric field, \( k \) is the wavevector of the radiation along the propagation direction, and \( \omega \) is the angular frequency of oscillation of the radiation.
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If the energy, \( \mathcal{E} \) is in keV, the relationship among these quantities is given by:

\[ \hbar \omega = \hbar \nu = \mathcal{E}, \lambda \nu = c \lambda = \frac{hc}{\mathcal{E}} = \left(4.1357 \times 10^{-15} \text{eV} \cdot \text{s}\right) \left(2.9979 \times 10^8 \text{m/s}\right)/\mathcal{E} = \left(4.1357 \times 10^{-18} \text{keV} \cdot \text{s}\right) \left(2.9979 \times 10^{18} \text{˚A/s}\right)/\mathcal{E} = 12.398 \text{˚A/keV} \]
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\[ = (4.1357 \times 10^{-18} \text{ keV} \cdot \text{s})(2.9979 \times 10^{18} \text{ Å/s})/\mathcal{E} \]
\[ = 12.398 \text{ Å} \cdot \text{keV}/\mathcal{E} \quad \text{to give units of Å} \]
Interactions of x-rays with matter

For the purposes of this course, we care most about the interactions of x-rays with matter.

There are four basic types of such interactions:

1. Elastic scattering
2. Inelastic scattering
3. Absorption
4. Pair production

We will only discuss the first three.
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The typical scattering geometry is

\[ k \rightarrow k' \rightarrow Q \]

where an incident x-ray of wave number \( k \) scatters elastically from an electron to \( k' \) resulting in a scattering vector \( Q \) or in terms of momentum transfer:

\[ \hbar Q = \hbar k - \hbar k' \]
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Start with the scattering from a single electron, then build up to more complexity.
Thomson scattering

Assumptions:

incident x-ray plane wave

electron is a point charge

scattering is elastic

\[ \text{scattered intensity} \propto \frac{1}{R^2} \]

The electron is exposed to the incident electric field \( E_{\text{in}}(t') \) and is accelerated.

The acceleration of the electron, \( a_x(t') \), results in the radiation of a spherical wave with the same frequency.

The observer at \( R \) "sees" a scattered electric field \( E_{\text{rad}}(R, t) \) at a later time \( t = t' + R/c \).

Using this, calculate the elastic scattering cross-section.
Thomson scattering

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![Diagram of Thomson scattering](image)
Thomson scattering

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Thomson scattering

\[ E_{\text{rad}}(R, t) = -\frac{e}{4\pi \epsilon_0 c^2 R} a_x(t') \sin \Psi \]
Thomson scattering

\[ E_{rad}(R, t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} a_x(t') \sin \Psi \]

where \( t' = t - \frac{R}{c} \)
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\[ \frac{E_{\text{rad}}(R, t)}{E_{\text{in}}} = -\frac{e^2}{4\pi \epsilon_0 m c^2} \frac{e^{i\omega R/c}}{R} \sin \Psi \]
Thomson scattering

\[
E_{\text{rad}}(R, t) = -\frac{e}{4\pi\varepsilon_0 c^2 R} \frac{e}{m} E_{\text{in}} e^{i\omega R/c} \sin \Psi
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\[
\frac{E_{\text{rad}}(R, t)}{E_{\text{in}}} = -\frac{e^2}{4\pi\varepsilon_0 mc^2} \frac{e^{i\omega R/c}}{R} \sin \Psi \quad \text{but} \quad k = \frac{\omega}{c}
\]
Thomson scattering

\[ \frac{E_{\text{rad}}(R, t)}{E_{\text{in}}} = - \frac{e^2}{4\pi \epsilon_0 mc^2} \frac{e^{ikR}}{R} \sin \Psi = -r_0 \frac{e^{ikR}}{R} \sin \Psi \]
Thomson scattering

\[ E_{\text{rad}}(R, t) \]

\[ = - \frac{e^2}{4\pi \epsilon_0 mc^2} \frac{e^{ikR}}{R} \sin \Psi \]

\[ = -r_0 \frac{e^{ikR}}{R} \sin \Psi \]

\[ r_0 = \frac{e^2}{4\pi \epsilon_0 mc^2} = 2.82 \times 10^{-5} \text{Å} \]
Scattering cross-section

The cross-section of incoming beam is $A_0$. The cross-section of scattered beam (into detector) is $R^2 \Delta \Omega \Phi_0 \equiv I_0 A_0 = c |E_{in}|^2 \hbar \omega$.

The intensity of scattered beam is $I_{sc} \propto c (R^2 \Delta \Omega) |E_{rad}|^2 |E_{in}|^2 R^2 \Delta \Omega$. 

C. Segre (IIT)

PHYS 570 - Spring 2018

January 09, 2018 18 / 20
Scattering cross-section

Detector of solid angle $\Delta \Omega$ at a distance $R$ from electron
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Cross-section of incoming beam is $A_0$

$$\Phi_0 \equiv \frac{I_0}{A_0} = c \frac{|E_{in}|^2}{\hbar \omega}$$
Scattering cross-section

Detector of solid angle $\Delta\Omega$ at a distance $R$ from electron
Cross-section of incoming beam is $A_0$

$$\Phi_0 \equiv \frac{l_0}{A_0} = c \frac{|E_{in}|^2}{\hbar \omega}$$

$$I_{sc} \propto c (R^2 \Delta \Omega) \frac{|E_{rad}|^2}{\hbar \omega}$$
Scattering cross-section

Detector of solid angle $\Delta \Omega$ at a distance $R$ from electron
Cross-section of incoming beam is $A_0$
Cross section of scattered beam (into detector) is $R^2 \Delta \Omega$

$$\Phi_0 \equiv \frac{l_0}{A_0} = c \frac{|E_{in}|^2}{\hbar \omega}$$

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$$\frac{I_{sc}}{I_0} = \frac{|E_{rad}|^2}{|E_{in}|^2} R^2 \Delta \Omega$$
Scattering cross-section

Differential cross-section is obtained by normalizing

$$d\sigma d\Omega = I_{sc} \Phi_0 \Delta\Omega = I_{sc} \left( \frac{I_0}{A_0} \right) \Delta\Omega = |E_{rad}|^2 |E_{in}|^2 \frac{R^2}{r_0^2} \sin^2 \Psi = -r_0 e^{ikR} R \left| \hat{\epsilon} \cdot \hat{\epsilon}' \right| = -r_0 e^{ikR} R \left| \cos \left( \frac{\pi}{2} - \Psi \right) \right| = -r_0 e^{ikR} R \sin \Psi$$
Scattering cross-section

Differential cross-section is obtained by normalizing

\[
\frac{d\sigma}{d\Omega} = I_{sc} \Phi_0 \Delta \Omega = I_{sc} \left( \frac{I_0}{A_0} \right) \Delta \Omega = |E_{rad}|^2 |E_{in}|^2 R^2 = r_0^2 \sin^2 \Psi
\]

\[
E_{rad} E_{in} = -r_0 e^{i k R} R |\hat{\epsilon} \cdot \hat{\epsilon}'| = -r_0 e^{i k R} R \cos(\pi/2 - \Psi) |\sin \Psi|
\]
Differential cross-section is obtained by normalizing

\[
\frac{d\sigma}{d\Omega} = \frac{I_{sc}}{\Phi_0 \Delta \Omega}
\]
Scattering cross-section

Differential cross-section is obtained by normalizing

\[ \frac{d\sigma}{d\Omega} = \frac{l_{sc}}{\Phi_0 \Delta\Omega} = \frac{l_{sc}}{(l_0/A_0) \Delta\Omega} \]
Scattering cross-section

Differential cross-section is obtained by normalizing

\[
\frac{d\sigma}{d\Omega} = \frac{l_{sc}}{\Phi_0 \Delta \Omega} = \frac{l_{sc}}{(I_0/A_0) \Delta \Omega} = \frac{|E_{rad}|^2}{|E_{in}|^2} R^2
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\]
Scattering cross-section

Differential cross-section is obtained by normalizing

\[
\frac{d\sigma}{d\Omega} = \frac{I_{sc}}{\Phi_0 \Delta\Omega} = \frac{I_{sc}}{(I_0/A_0) \Delta\Omega} = \frac{|E_{rad}|^2}{|E_{in}|^2} R^2
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Scattering cross-section

Differential cross-section is obtained by normalizing

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Total cross-section

Integrate to obtain the total Thomson scattering cross-section from an electron.

σ = \frac{8}{3} \pi r_0^2 = 0.665 \times 10^{-24} \text{ cm}^2 = 0.665 \text{ barn}

Polarization factor = \begin{cases} 
1 & \sin^2 \Psi \\
\frac{1}{2} (1 + \sin^2 \Psi) & \end{cases}
Integrate to obtain the total Thomson scattering cross-section from an electron.

\[
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\]
Integrate to obtain the total Thomson scattering cross-section from an electron.

\[ \sigma = \frac{8\pi}{3} r_0^2 \]
\[ = 0.665 \times 10^{-24} \text{ cm}^2 \]
\[ = 0.665 \text{ barn} \]
Integrate to obtain the total Thomson scattering cross-section from an electron. If displacement is in vertical direction, $\sin \Psi$ term is replaced by unity and if the source is unpolarized, it is a combination.

\[
\sigma = \frac{8\pi}{3} r_0^2
\]

\[
= 0.665 \times 10^{-24} \text{ cm}^2
\]

\[
= 0.665 \text{ barn}
\]

Polarization factor:

\[
\frac{1}{2} \left( 1 + \sin^2 \Psi \right)
\]