Today's outline - August 28, 2024

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- The bending magnet source

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  - Curved arc emission

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  - Characteristic energy

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Reading Assignment: Chapter 2.5–2.6

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Reading Assignment: Chapter 2.5–2.6

Homework Assignment #01:
Chapter 2: 2,3,5,6,8
due Wednesday, September 04, 2024

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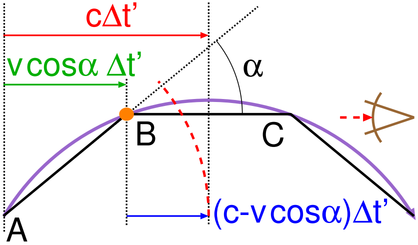
Reading Assignment: Chapter 2.5–2.6

Homework Assignment #01:
Chapter 2: 2,3,5,6,8
due Wednesday, September 04, 2024

Homework Assignment #02:
Problems on Canvas
due Monday, September 16, 2024

Segmented arc review

The first approximation to a bending magnet source is the segmented arc



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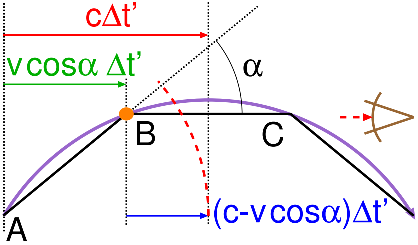

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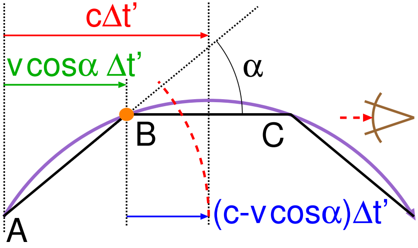

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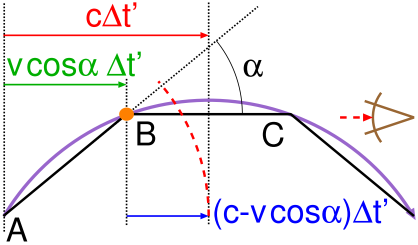

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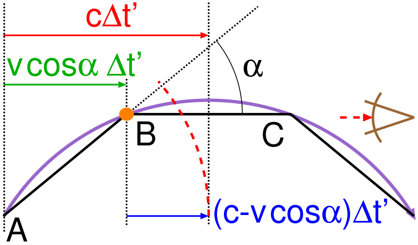

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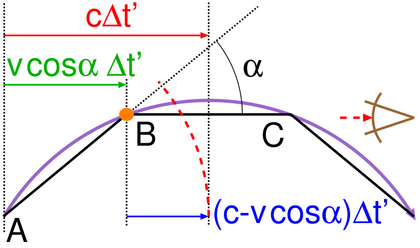

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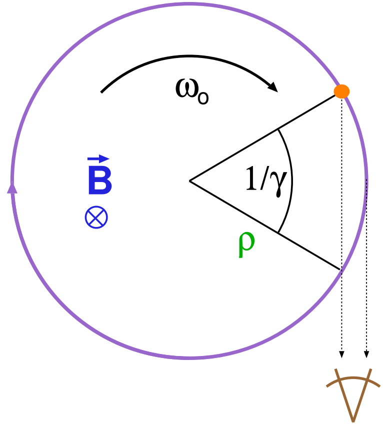
$$\frac{\Delta t}{\Delta t'} = (1 - \beta \cos\alpha)$$

Curved arc emission



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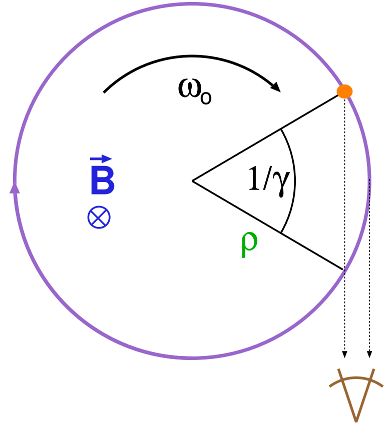

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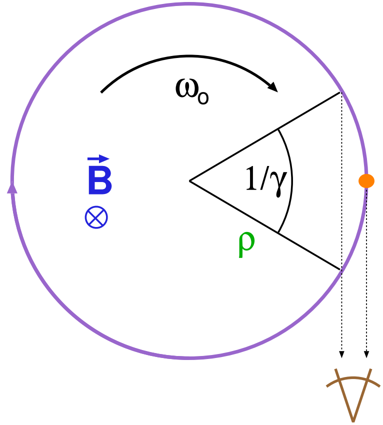

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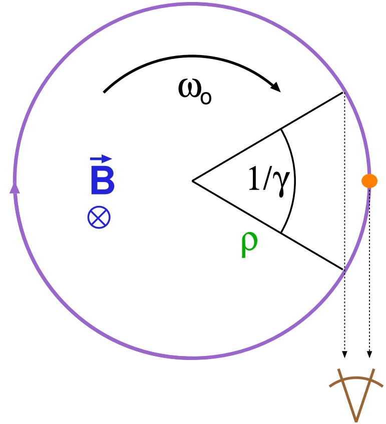

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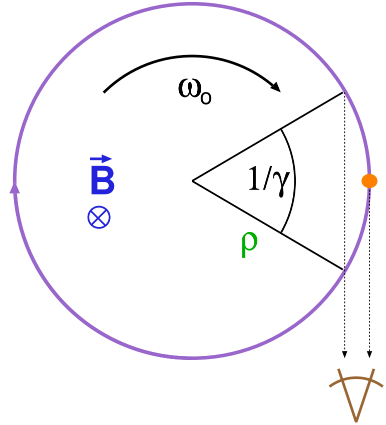

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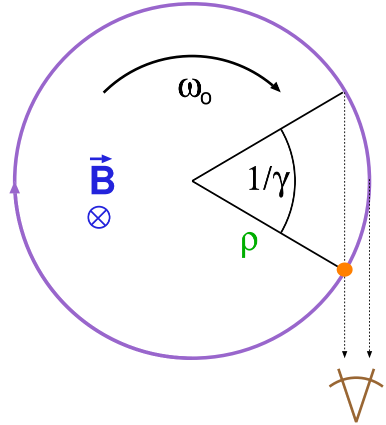

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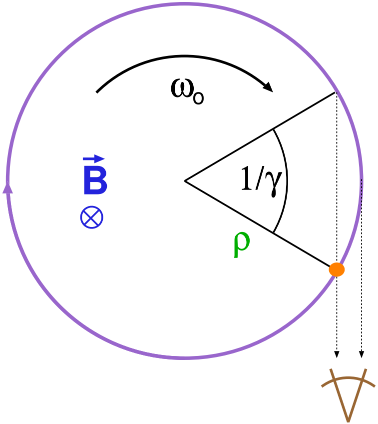

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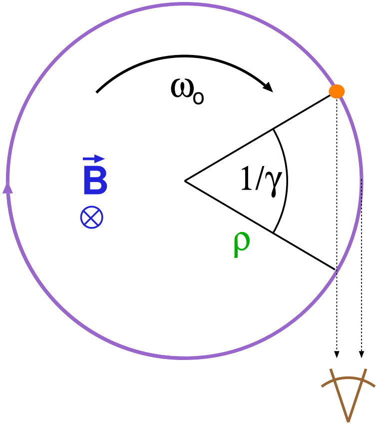
Electron bending radius



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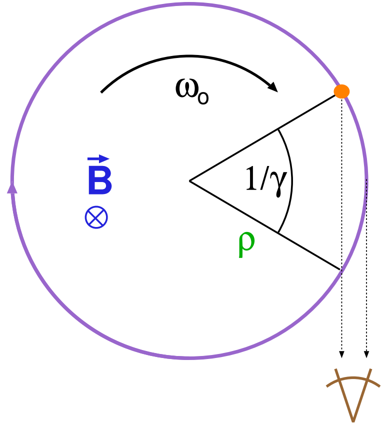

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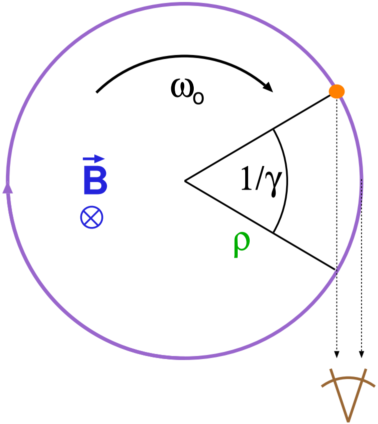

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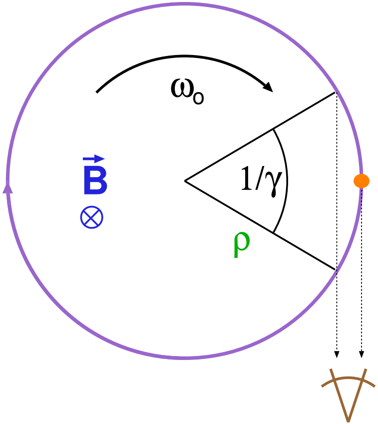

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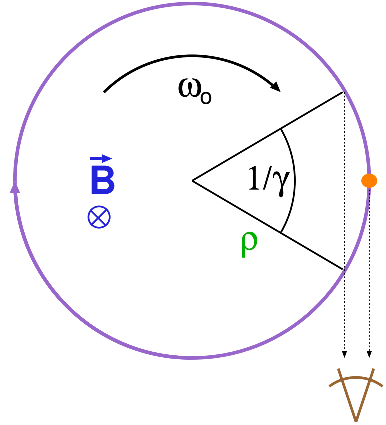

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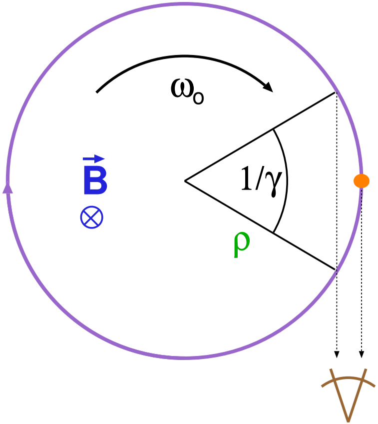

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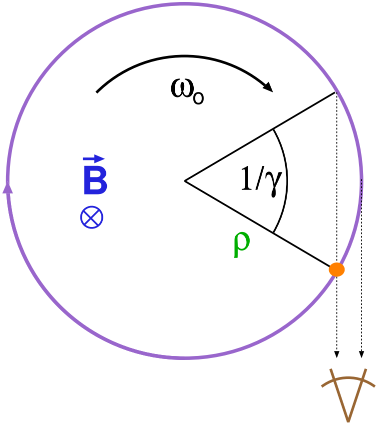

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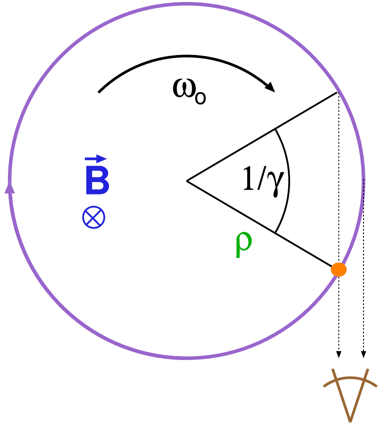

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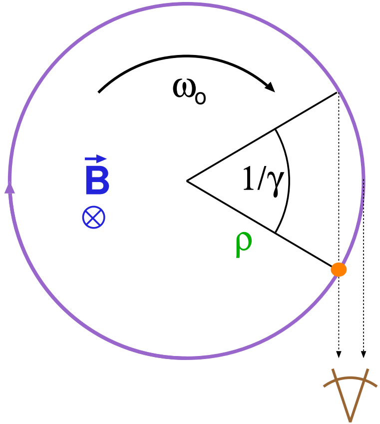

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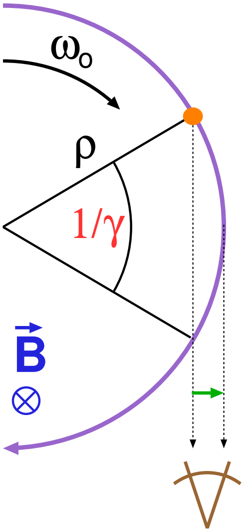
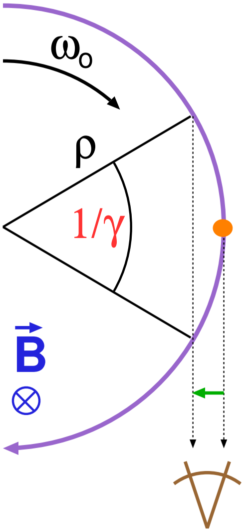
Curved arc emission



The observer, looking in the plane of the circular trajectory,

Curved arc emission



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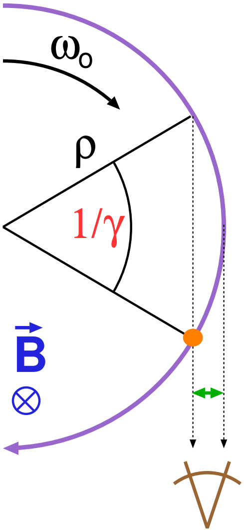

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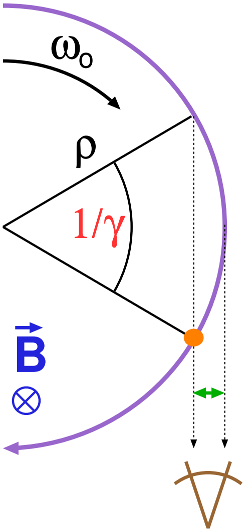

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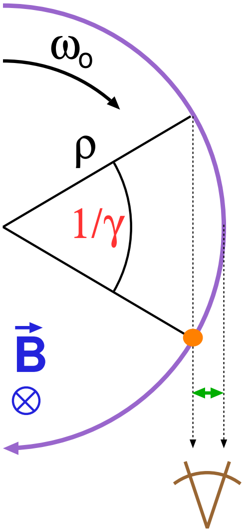

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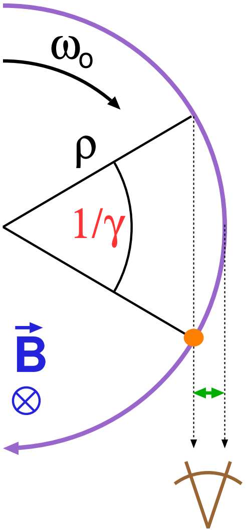

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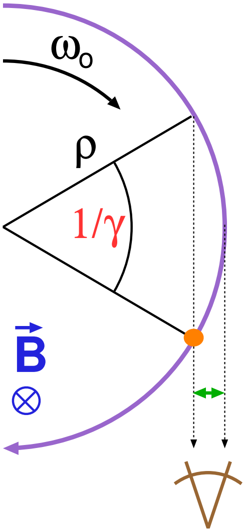

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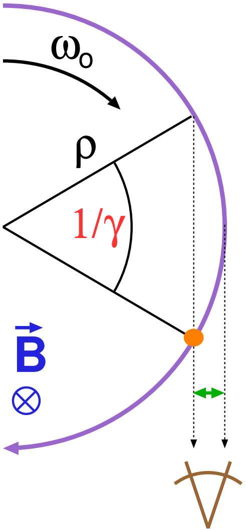

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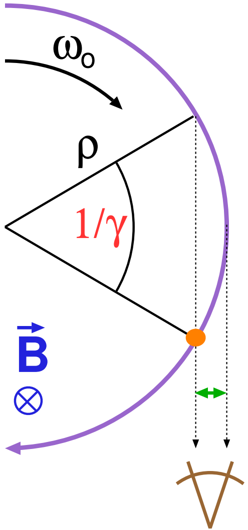

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The Fourier transform of this pulse is the spectrum of the radiation from the bending magnet.

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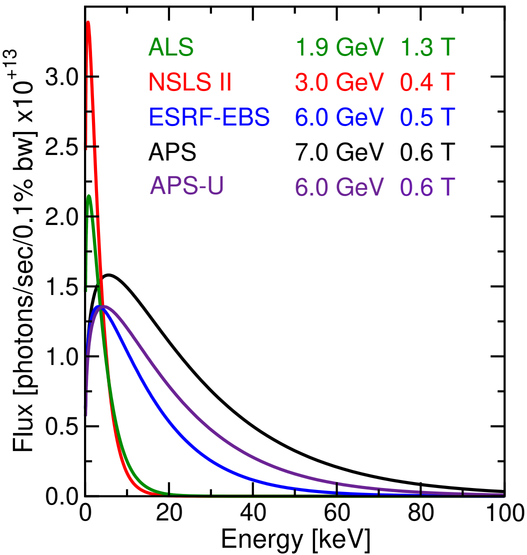
converting to storage ring units

$$\mathcal{E}_c[\text{keV}] = 0.665\mathcal{E}^2[\text{GeV}]B[\text{T}]$$

Bending magnet spectrum

When the radiation pulse time is Fourier transformed, we obtain the spectrum of a bending magnet.



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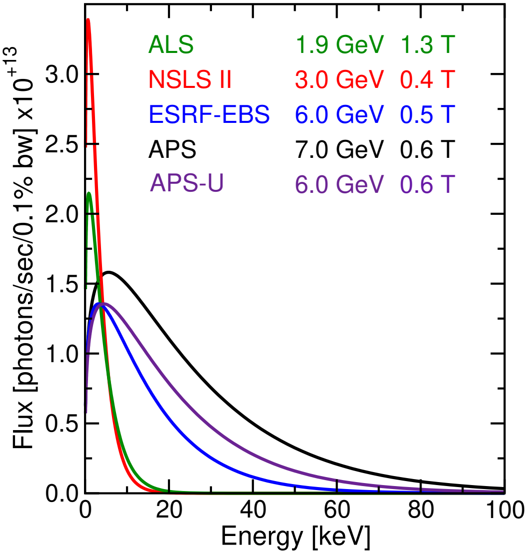

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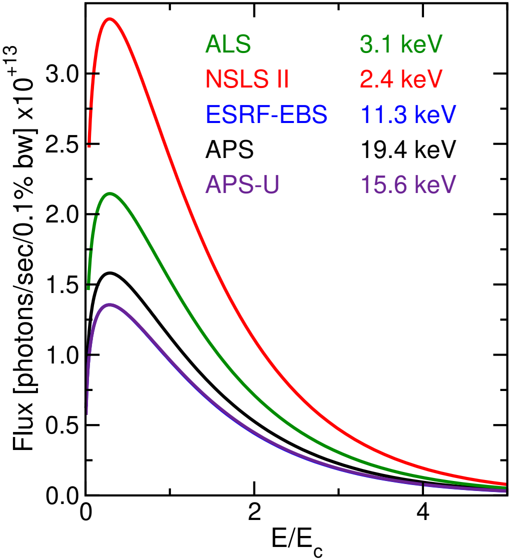

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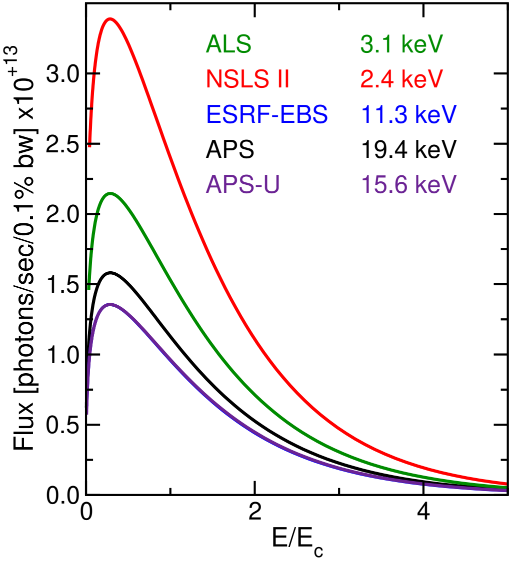
$$1.33 \times 10^{13} \mathcal{E}^2 I \left(\frac{\omega}{\omega_c}\right)^2 K_{2/3}^2 \left(\frac{\omega}{2\omega_c}\right)$$

where $K_{2/3}$ is a modified Bessel function of the second kind.



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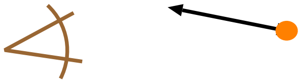
$$P = 1.266(6\text{GeV})^2(0.8\text{T})^2(1.24 \times 10^{-3}\text{m})(0.2\text{A}) = 7.3\text{W}$$

Polarization

A bending magnet also produces circularly polarized radiation



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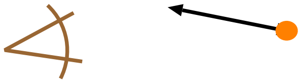

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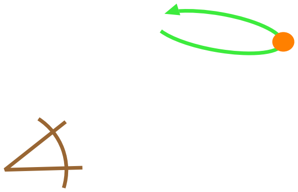

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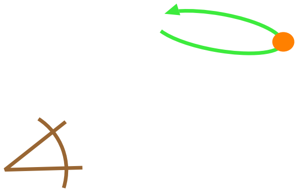

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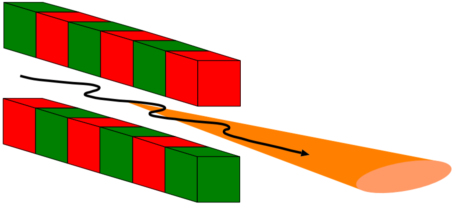
The result is circularly polarized radiation above and below the on-axis radiation.

Wigglers and undulators

Wiggler



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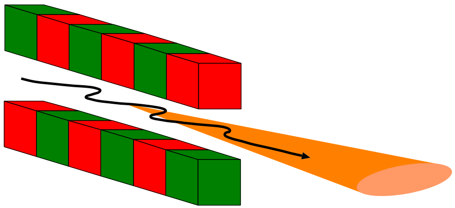
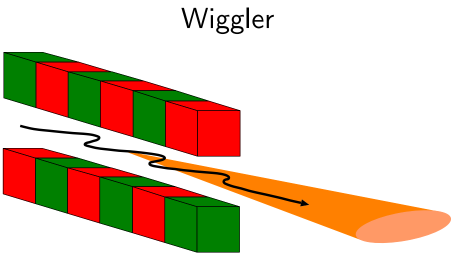


Like bending magnet except:

Wigglers and undulators



Like bending magnet except:

- larger $\vec{B} \rightarrow$ higher E_c

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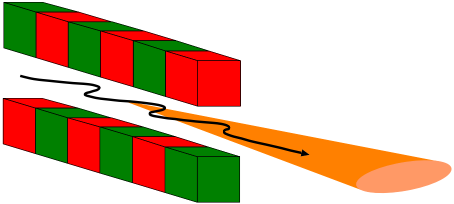

Like bending magnet except:

- larger $\vec{B} \rightarrow$ higher E_c
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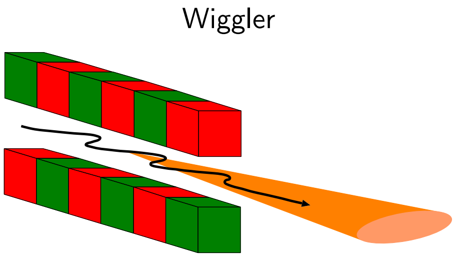

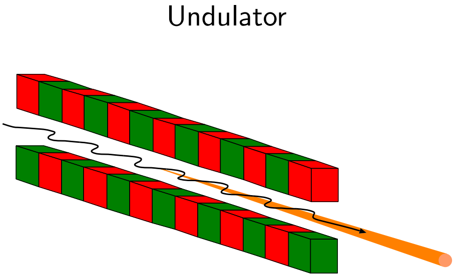

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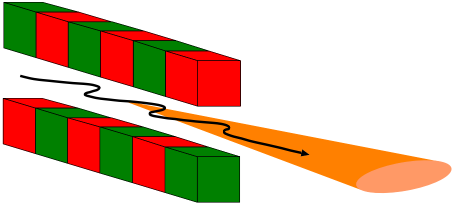
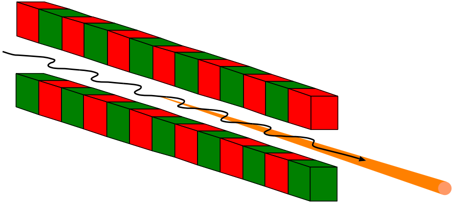


Undulator



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Different from bending magnet:

Wigglers and undulators

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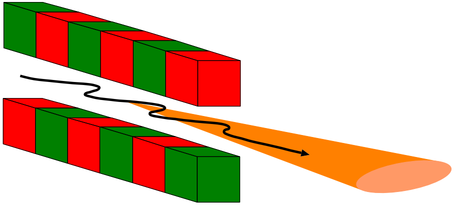
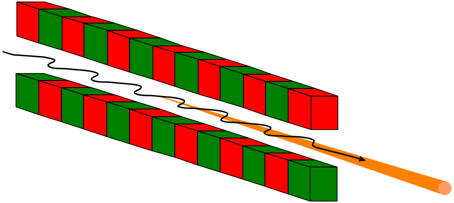


Undulator



Like bending magnet except:

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Different from bending magnet:

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Wigglers and undulators

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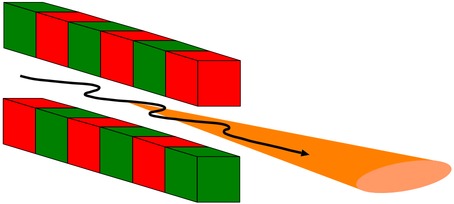

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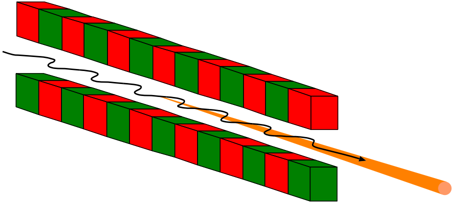

Like bending magnet except:

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Different from bending magnet:

- shallow bends $\rightarrow$ smaller source
- interference $\rightarrow$ peaked spectrum

Wiggler radiation

- The electron's trajectory through a wiggler can be considered as a series of short circular arcs, each like a bending magnet

$$Power[\text{kW}] = 1.266\mathcal{E}_e^2[\text{GeV}]B^2[\text{T}]L[\text{m}]I[\text{A}]$$

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- This results in a significantly higher power load on all downstream components

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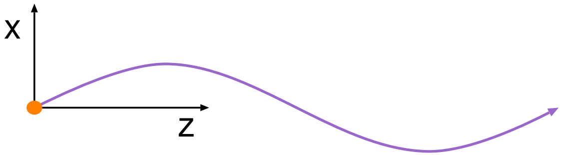
Undulator characterization



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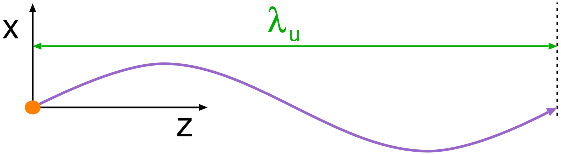

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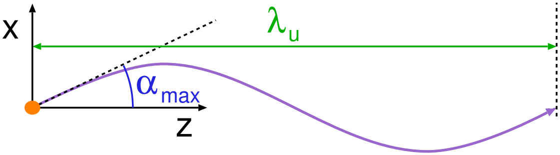

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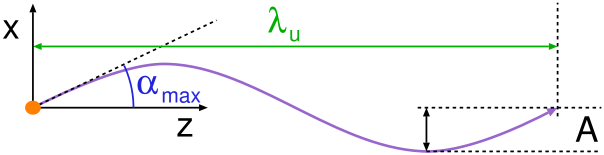
Undulator characterization



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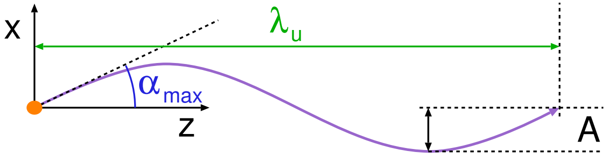

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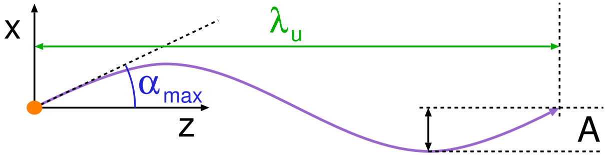

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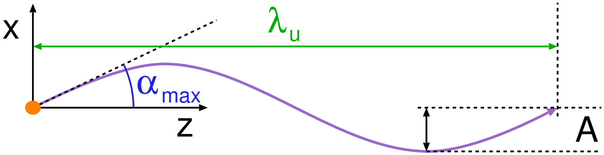

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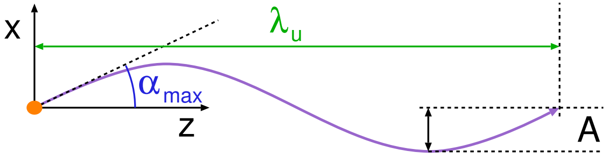

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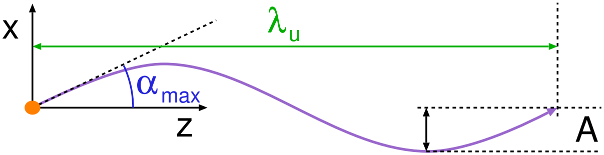

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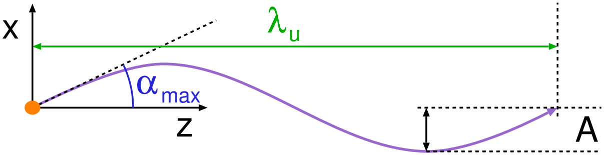

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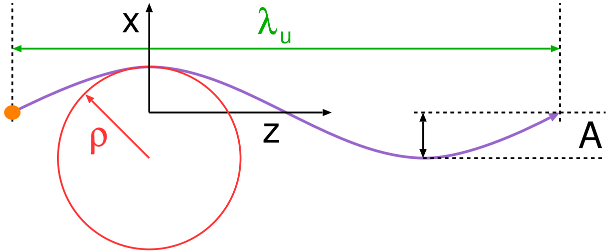
Circular path approximation



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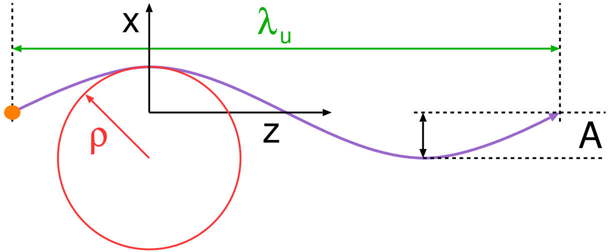
Circular path approximation



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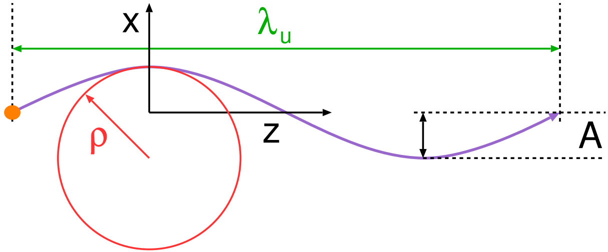

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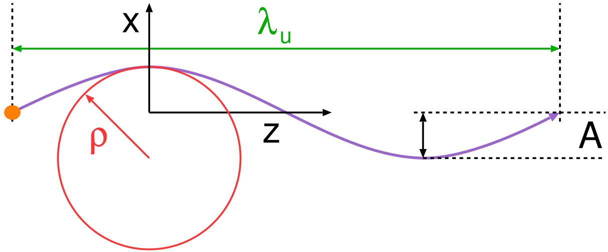

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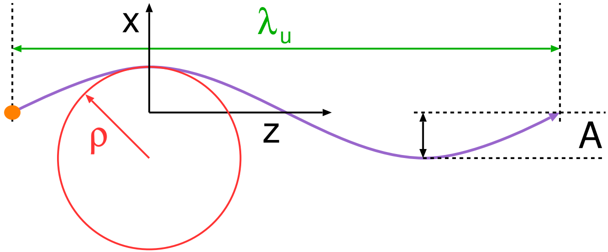

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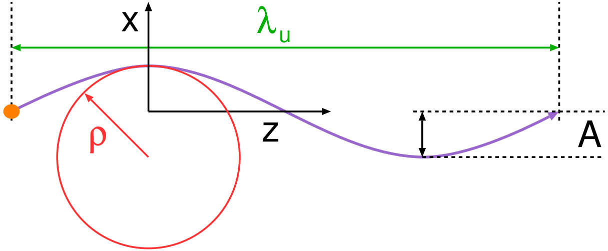

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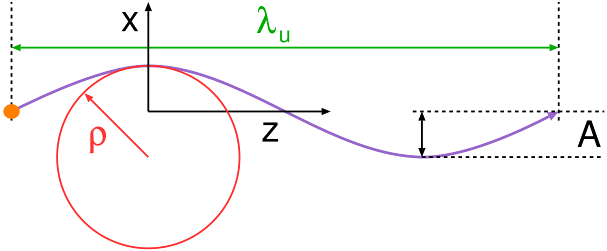

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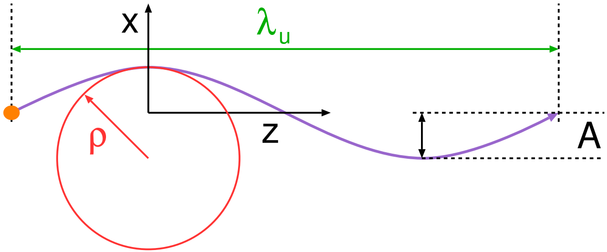

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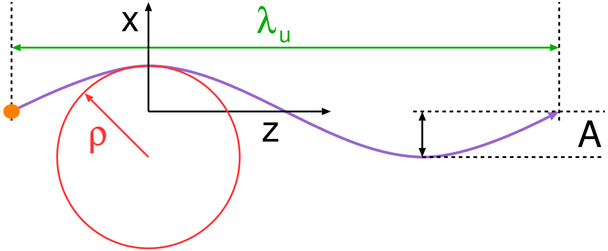

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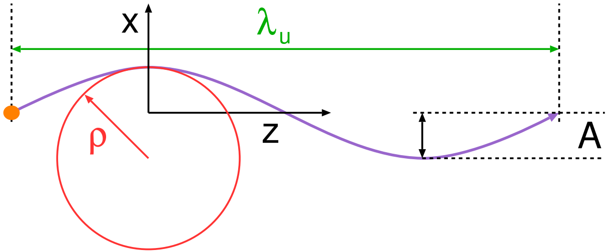

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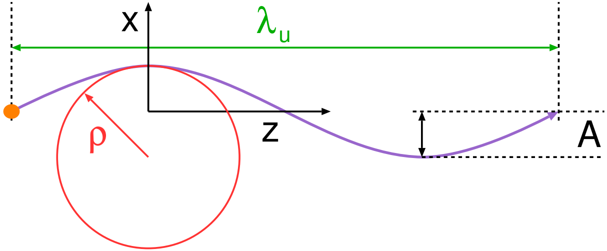

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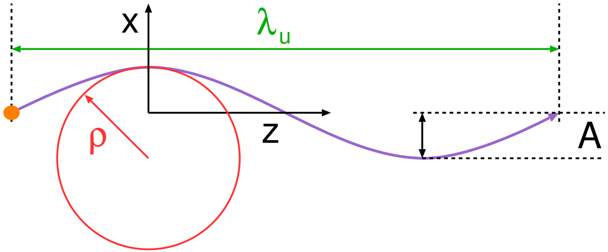

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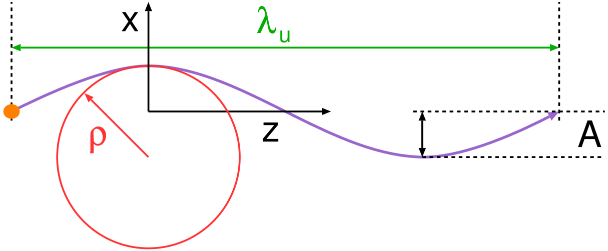

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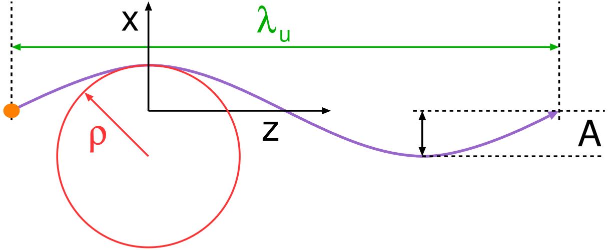

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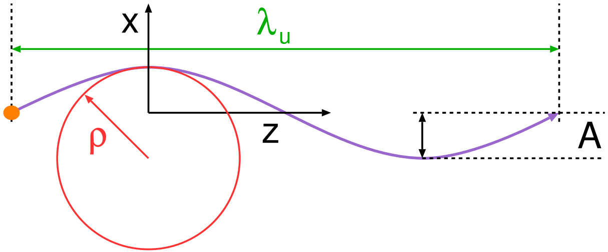

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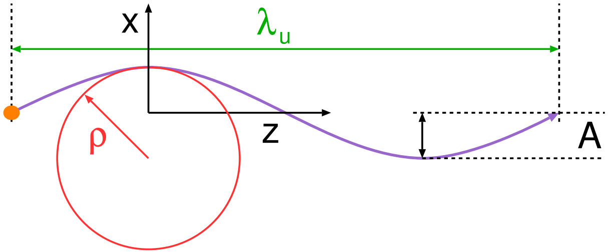

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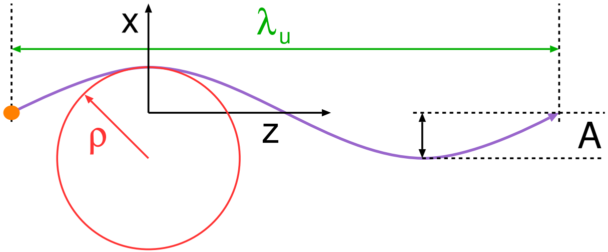

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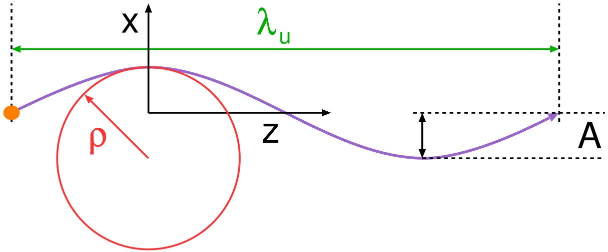

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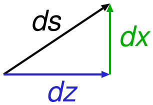
Electron path length

The displacement ds of the electron can be expressed in terms of the two coordinates, x and z as:



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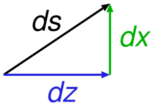

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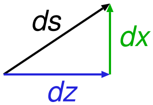

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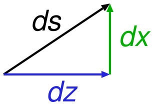

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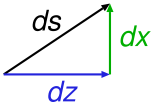

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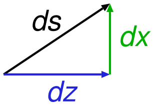

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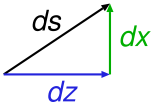

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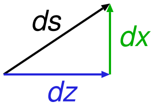

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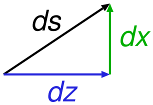

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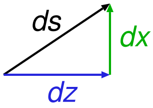

$$ds = \sqrt{(dx)^2 + (dz)^2} = \sqrt{1 + \left(\frac{dx}{dz}\right)^2} dz$$

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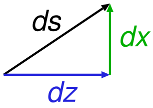

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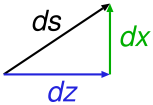

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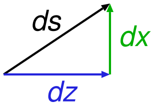

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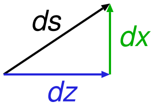

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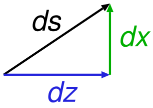

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Combining the above expressions yields

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For APS Undulator A, $\lambda_u = 3.3\text{cm}$ and $B_0 = 0.6\text{T}$ at closed gap, so

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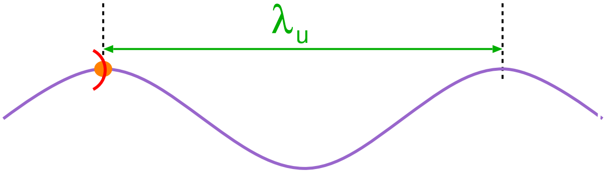
$$K = 0.934 \cdot 3.3[\text{cm}] \cdot 0.6[\text{T}] = 1.85$$

Undulator wavelength

Consider an electron traveling through the undulator and emitting radiation at the first maximum excursion from the center.



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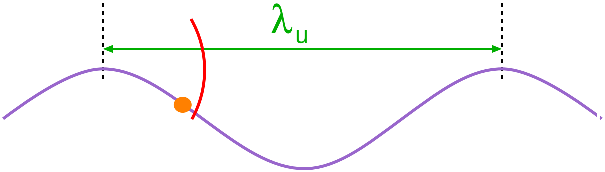

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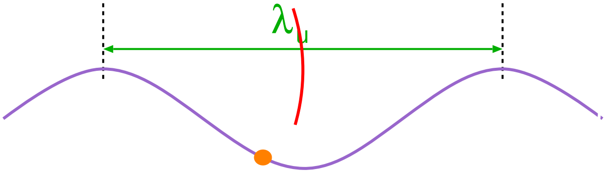

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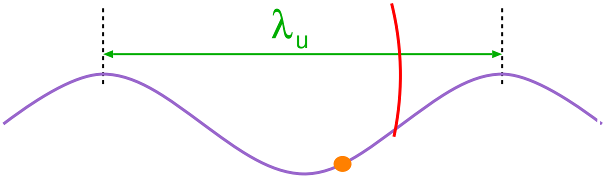

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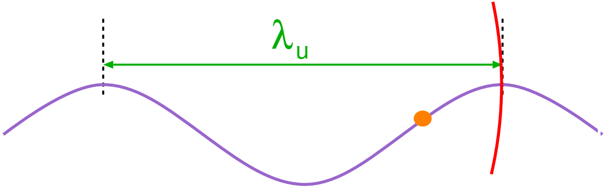

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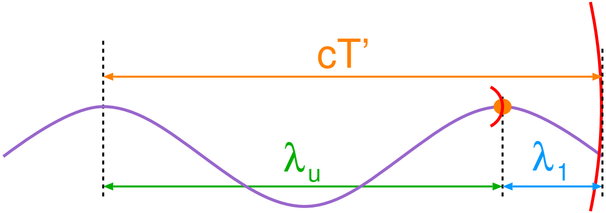

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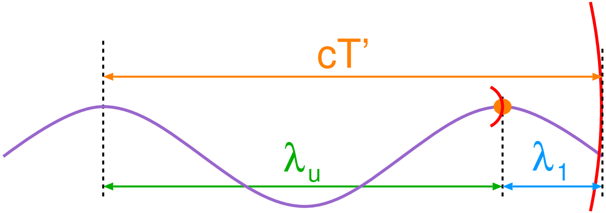

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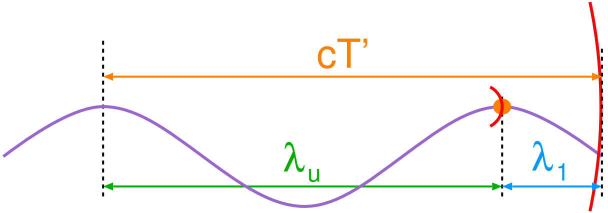

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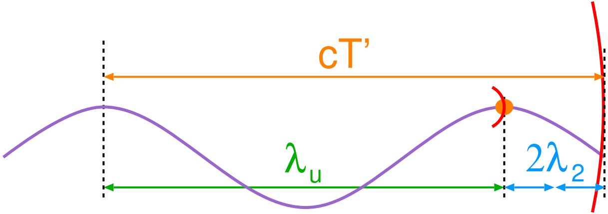

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The observer sees radiation with a compressed wavelength, along with harmonics which satisfy the same condition.

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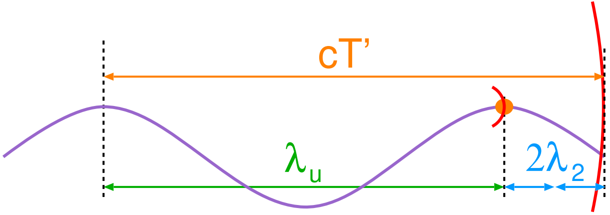

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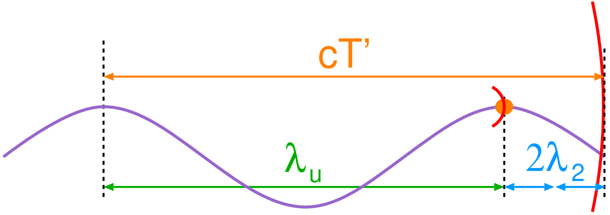

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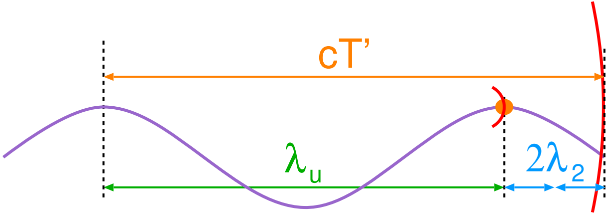

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$$\lambda_1 = cT' - \lambda_u \cos\theta$$

The fundamental wavelength

The fundamental wavelength emitted from the undulator depends on the photon propagation time, T'

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In a time T' the electron travels a distance $S\lambda_u$, so $T' = S\lambda_u/v$ and we know that

$$S \approx 1 + \frac{K^2}{4\gamma^2}$$

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Since γ is large, the maximum observation angle θ is small so

$$\lambda_1 = T' - \lambda_u \cos\theta = \frac{S\lambda_u}{v} - \lambda_u \cos\theta$$

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The fundamental wavelength

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The fundamental wavelength

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The fundamental wavelength

If we assume that $\beta \sim 1$ for these highly relativistic electrons

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for a typical undulator $\gamma \sim 10^4$, $K \sim 1$, and $\lambda_u \sim 2\text{cm}$ so we estimate

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