Today's outline - September 11, 2024

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- Refraction & reflection introduction

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Reading Assignment: Chapter 3.5–3.8

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- Refraction & reflection introduction
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- The Fresnel equations
- Reflectivity and Transmittivity
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Reading Assignment: Chapter 3.5–3.8

Homework Assignment #02:
Problems on September
due Monday, September 16, 2024

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- Refraction & reflection introduction
- Boundary conditions at an interface
- The Fresnel equations
- Reflectivity and Transmittivity
- Normalized q-coordinates

Reading Assignment: Chapter 3.5–3.8

Homework Assignment #02:
Problems on September
due Monday, September 16, 2024

Homework Assignment #03:
Chapter 3: 1,3,4,6,8
due Monday, September 30, 2024

Refraction & reflection of x-rays

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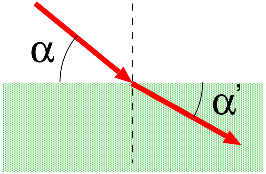

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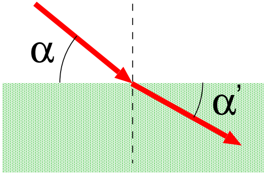

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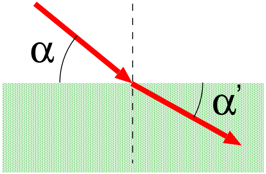

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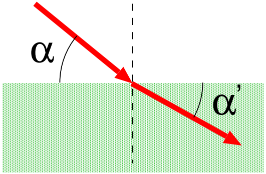

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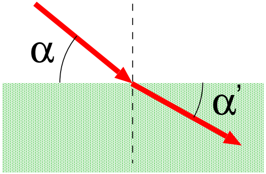

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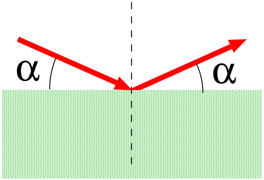

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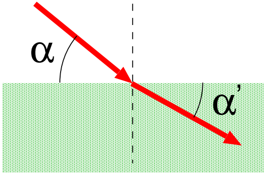

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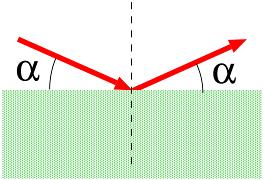

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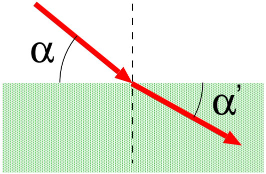

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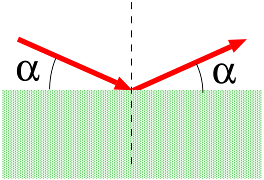

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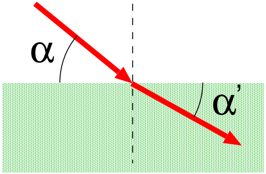

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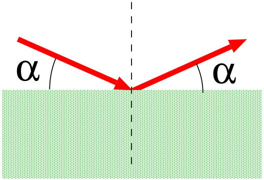

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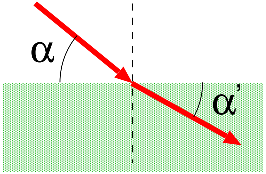

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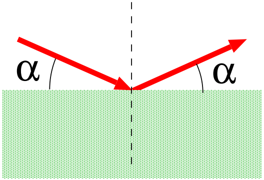

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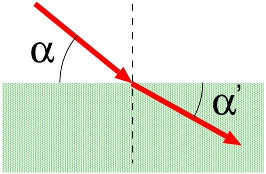

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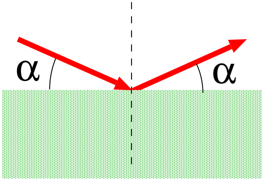

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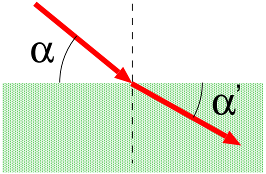

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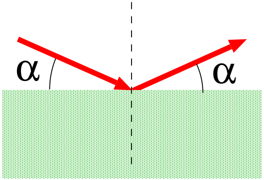

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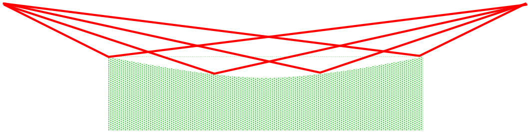
Uses of total external reflection



X-ray mirrors

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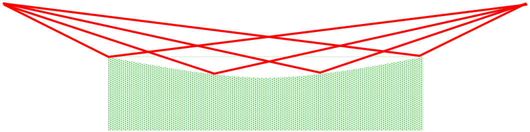

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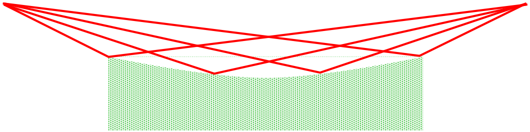

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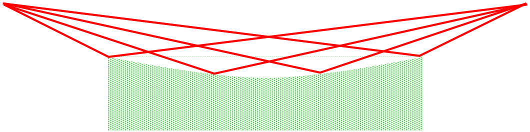

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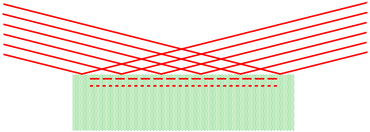

Evanscent wave experiments

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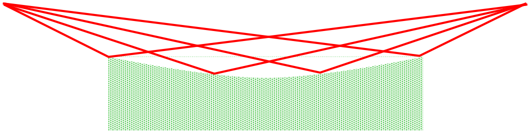

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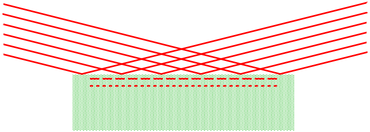

Evanscent wave experiments

- studies of surfaces

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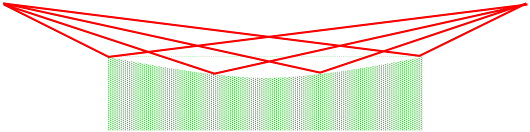

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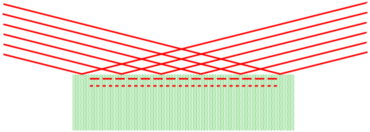

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Initially assume that all interfaces are perfectly flat and ignore all absorption processes.

Thin plate response - scattering approach

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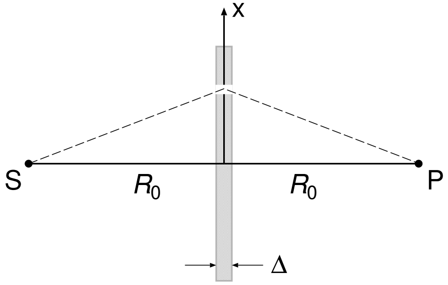

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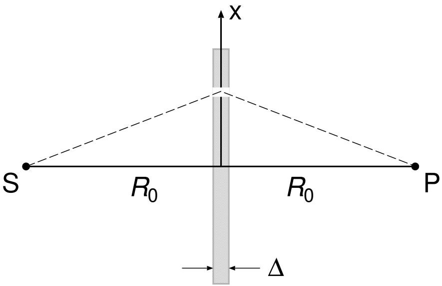

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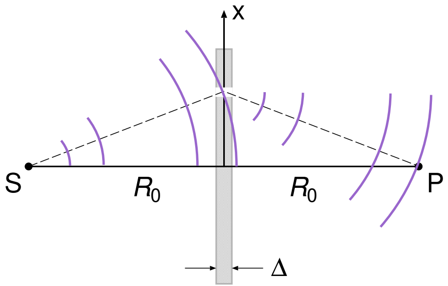

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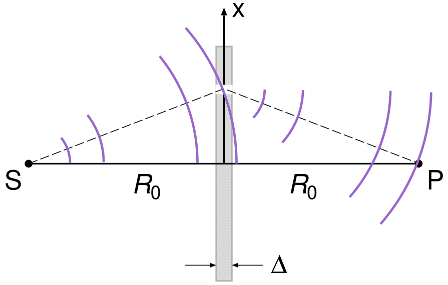

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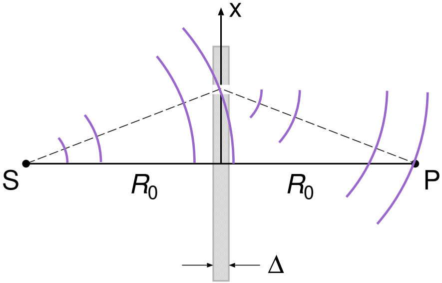

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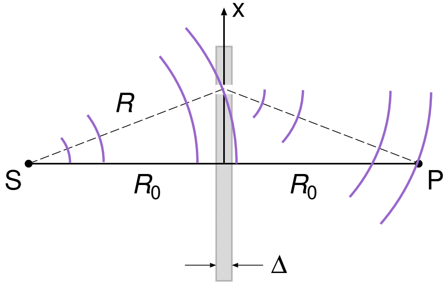

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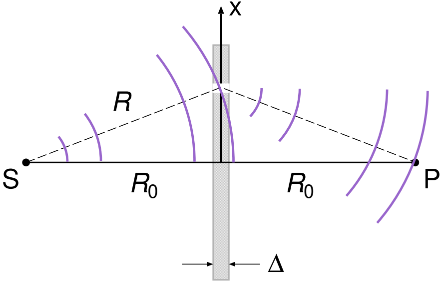

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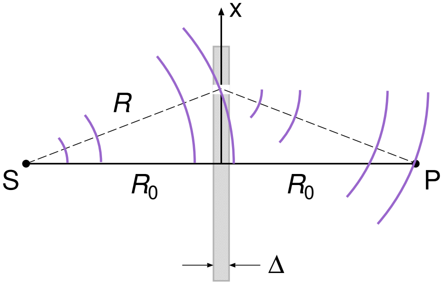

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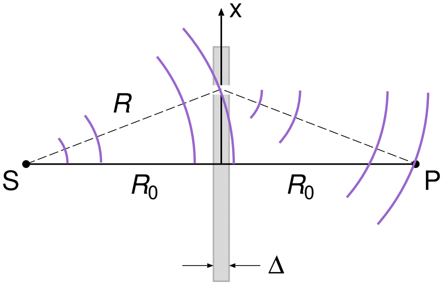

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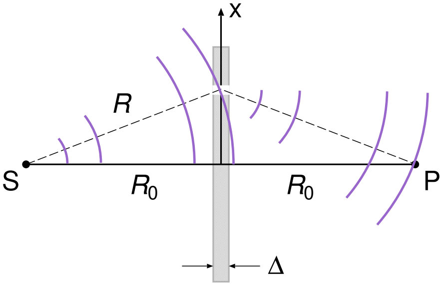

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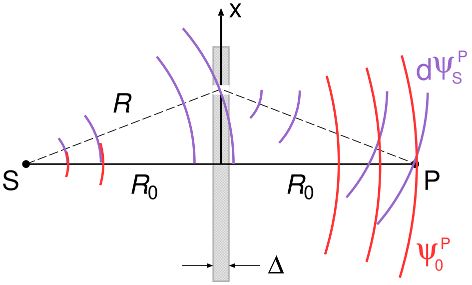

$$\phi(x,y) = 2k\frac{x^2+y^2}{2R_0^2} = \frac{x^2+y^2}{R_0^2}k$$

compared to a wave which travels directly along the z-axis.

Thin plate response - scattering approach

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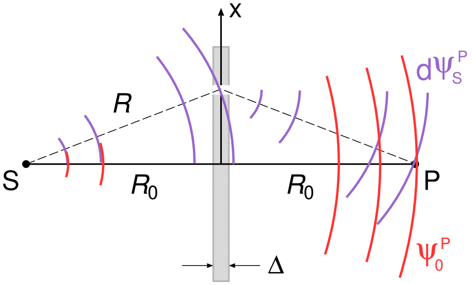

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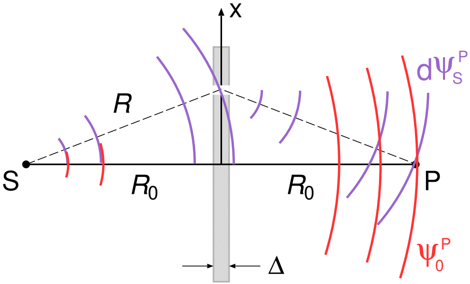

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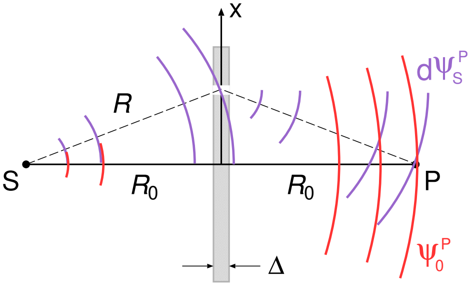

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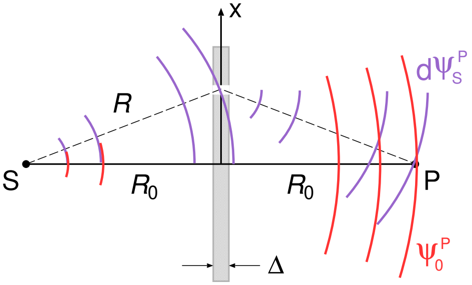

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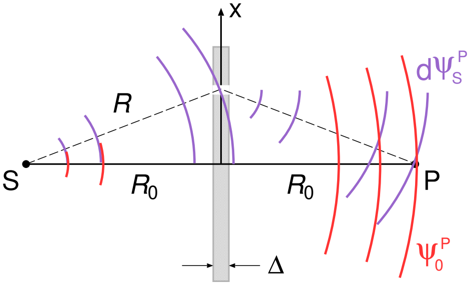

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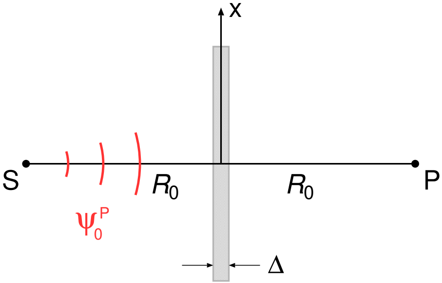

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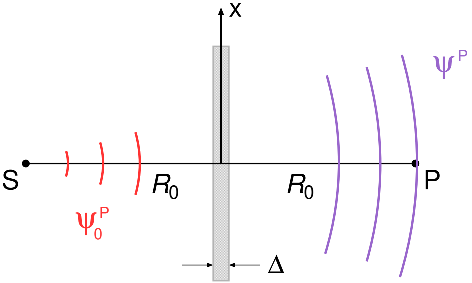

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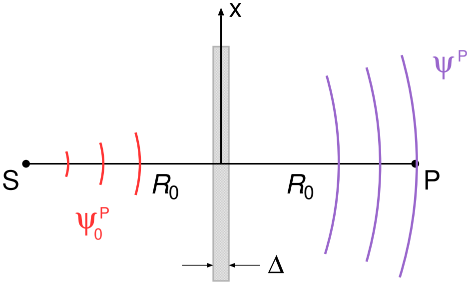

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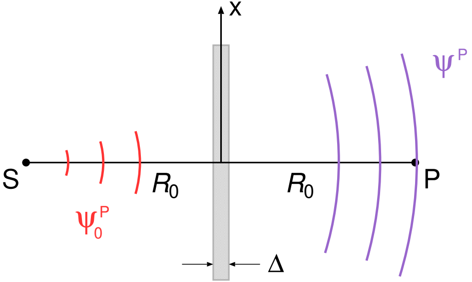

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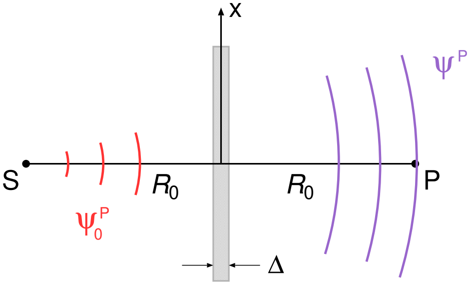

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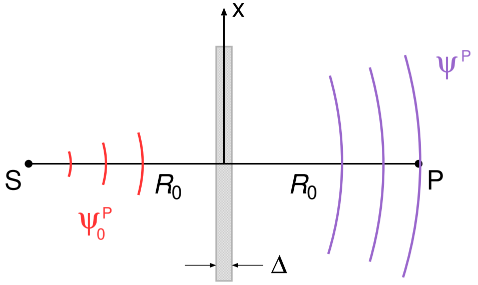

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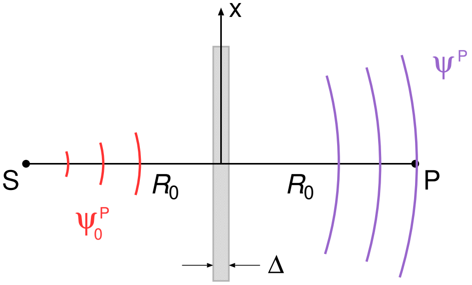

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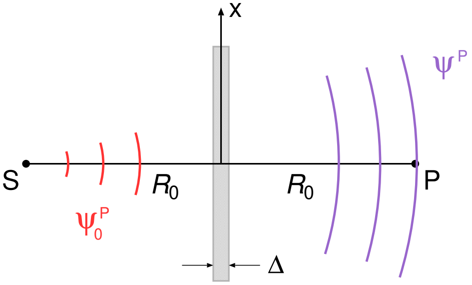

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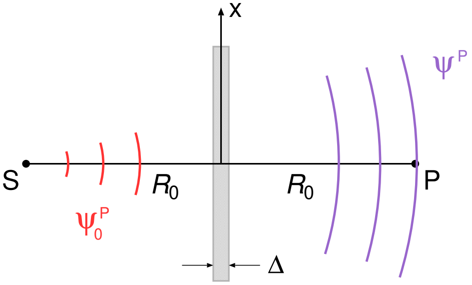

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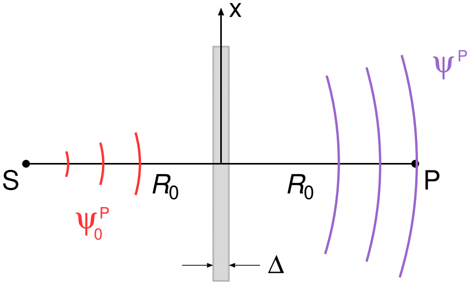

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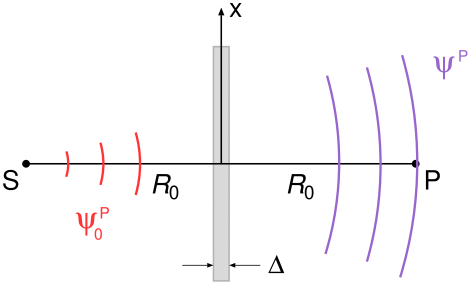

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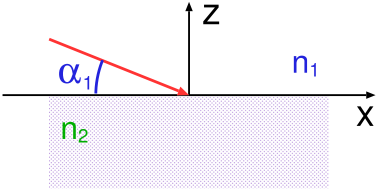
Index of refraction & critical angle

Consider an x-ray incident on an interface at angle α_1 to the surface



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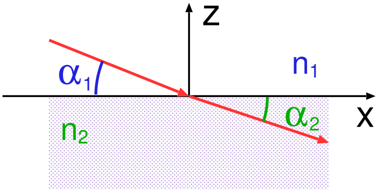

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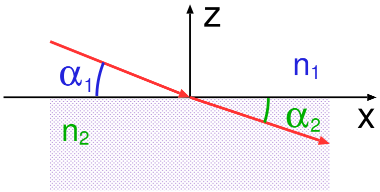
Applying Snell's Law



$$n_2 \cos \alpha_2 = n_1 \cos \alpha_1$$

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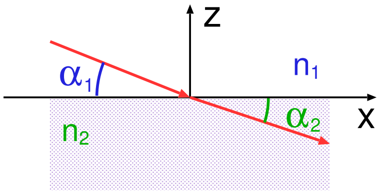

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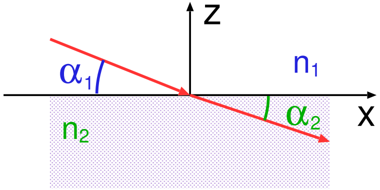

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If we now apply the known form of the index of refraction for the medium $(n_2 = 1 - \delta)$.

$$(1 - \delta)\cos\alpha_2 = \cos\alpha_1$$

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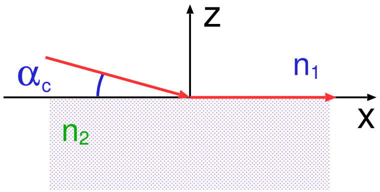

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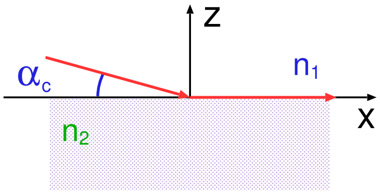

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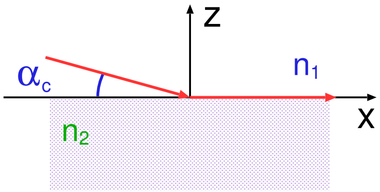

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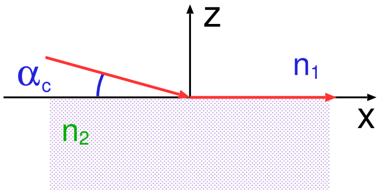

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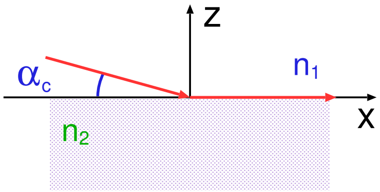

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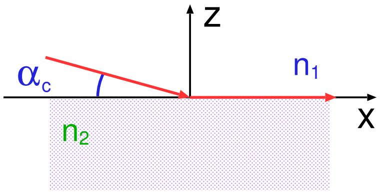

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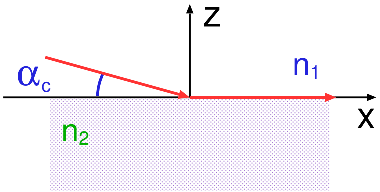

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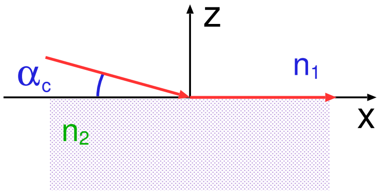

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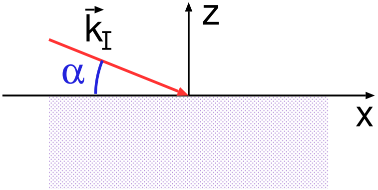
Electromagnetic boundary conditions

Maxwell's equations require that an electromagnetic wave and its derivative be continuous in all directions at any interface. This condition places restrictions on the waves which exist at any interface:



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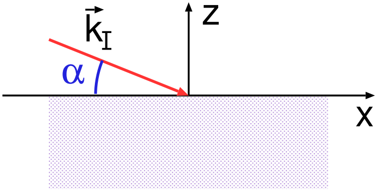

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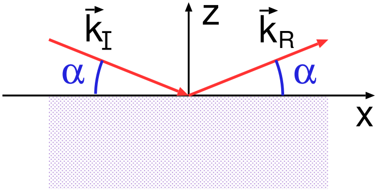

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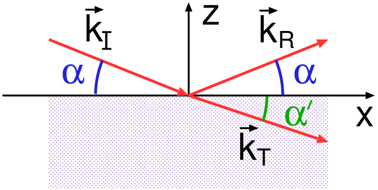

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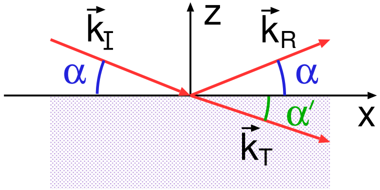

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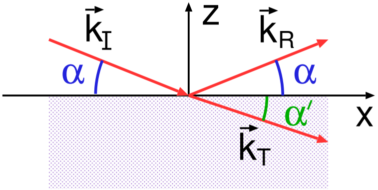

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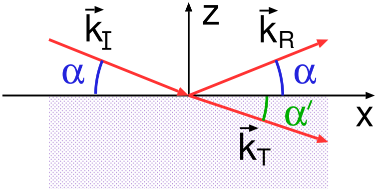

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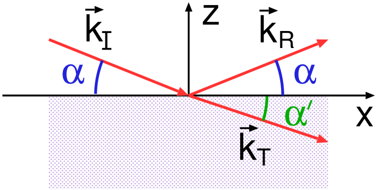

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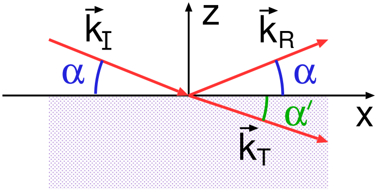

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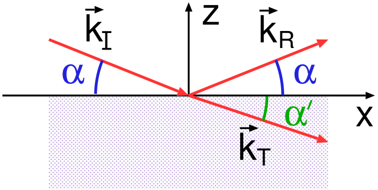

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Starting with the equation for the parallel projection of the field on the surface and noting that

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$$(a_I + a_R) n \cos\alpha' = (a_I + a_R) \cos\alpha$$
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$$1 - \frac{\alpha^2}{2} = (1 - \delta + i\beta)\left(1 - \frac{\alpha'^2}{2}\right)$$

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Recalling that $\alpha_c = \sqrt{2\delta}$

$$1 - \frac{\alpha^2}{2} = (1 - \delta + i\beta)\left(1 - \frac{\alpha'^2}{2}\right) \quad \longrightarrow \quad \alpha^2 = \alpha'^2 + \alpha_c^2 - 2i\beta$$

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$$r = \frac{a_R}{a_I} = \frac{\alpha - \alpha'}{\alpha + \alpha'}, \qquad t = \frac{a_T}{a_I} = \frac{2\alpha}{\alpha + \alpha'}$$

Reflectivity and transmittivity

r and t are called the reflection and transmission coefficients, respectively.

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In the z direction, the amplitude of the transmitted wave has two terms with the second one being the attenuation of the wave in the medium due to absorption. This attenuation is characterized by a quantity called the penetration depth, Λ.

$$a_T e^{ik\alpha' z} = a_T \; e^{ik\,\mathrm{Re}(\alpha')\mathrm{z}} \; e^{-k\,\mathrm{Im}(\alpha')\mathrm{z}}$$

$$\Lambda = \frac{1}{2k\,\mathrm{Im}(\alpha')}$$

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$$q = \frac{Q}{Q_c} \approx \frac{2k}{Q_c}\alpha \qquad q' = \frac{Q'}{Q_c} \approx \frac{2k}{Q_c}\alpha'$$

q is a convenient parameter to use because it is a combination of two parameters which are often varied in experiments, the angle of incidence α and the wavenumber (energy) of the x-ray, k.

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$$r = \frac{q - q'}{q + q'} \qquad t = \frac{2q}{q + q'}$$