Today's outline - October 23, 2024

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- Writing a General User Proposal

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- Dynamical theory and the Darwin curve

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Reading Assignment: Chapter 6.5; Chapter 7.1

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- Writing a General User Proposal
- Dynamical theory and the Darwin curve
- Extinction and absorption
- Perfect crystal integrated intensity

Reading Assignment: Chapter 6.5; Chapter 7.1

Homework Assignment #05:
Chapter 5: 1,2,7,9,10
due Monday, October 28, 2024

Today's outline - October 23, 2024

- Writing a General User Proposal
- Dynamical theory and the Darwin curve
- Extinction and absorption
- Perfect crystal integrated intensity

Reading Assignment: Chapter 6.5; Chapter 7.1

Homework Assignment #05:
Chapter 5: 1,2,7,9,10
due Monday, October 28, 2024

Homework Assignment #06:
Chapter 6: 1,6,7,8,9
due Monday, November 11, 2024

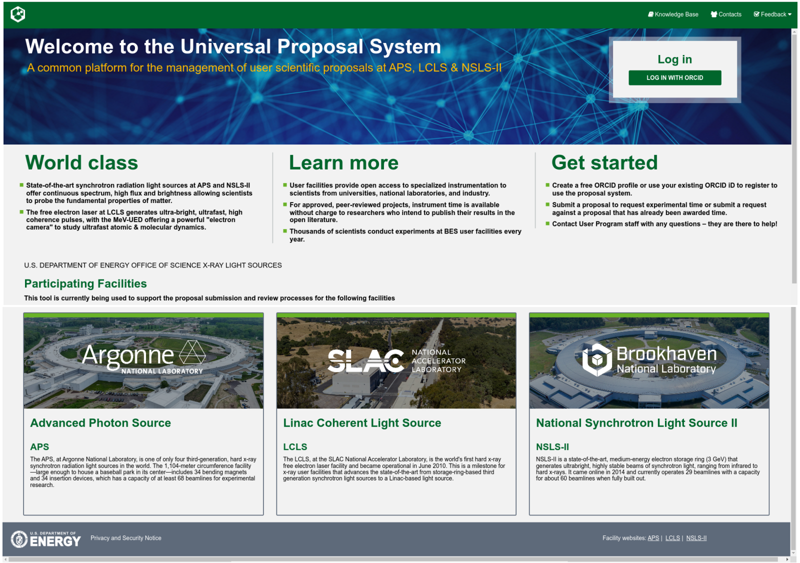
Writing a General User Proposal

1. Log into the UPS site
2. Start an APS general user proposal
3. Add an Abstract
4. Choose a beam line
5. Answer the 6 important questions

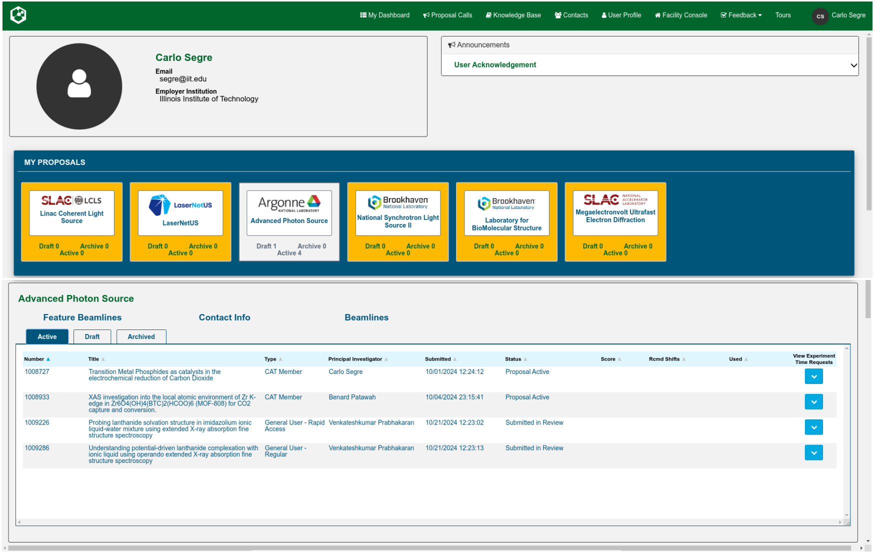
A tutorial can be found on the course home page

http://csrri.iit.edu/∼segre/phys570/24F/gu_proposal.html

Register & log into the UPS Portal



Select the APS



Start a proposal for and APS call

Argonne NATIONAL LABORATORY

Website
https://www.aps.anl.gov/

Location
9700 S. Cass Ave.
Lemont, IL 60439

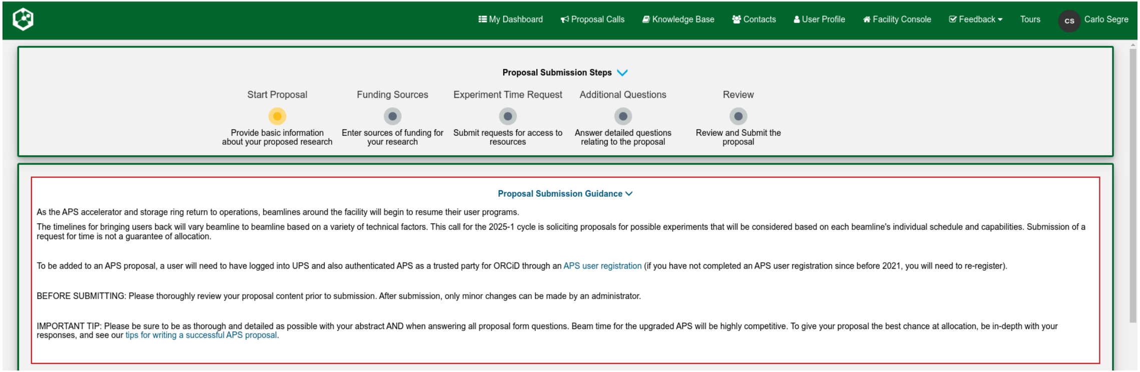
Phone
630-252-9090

Advanced Photon Source

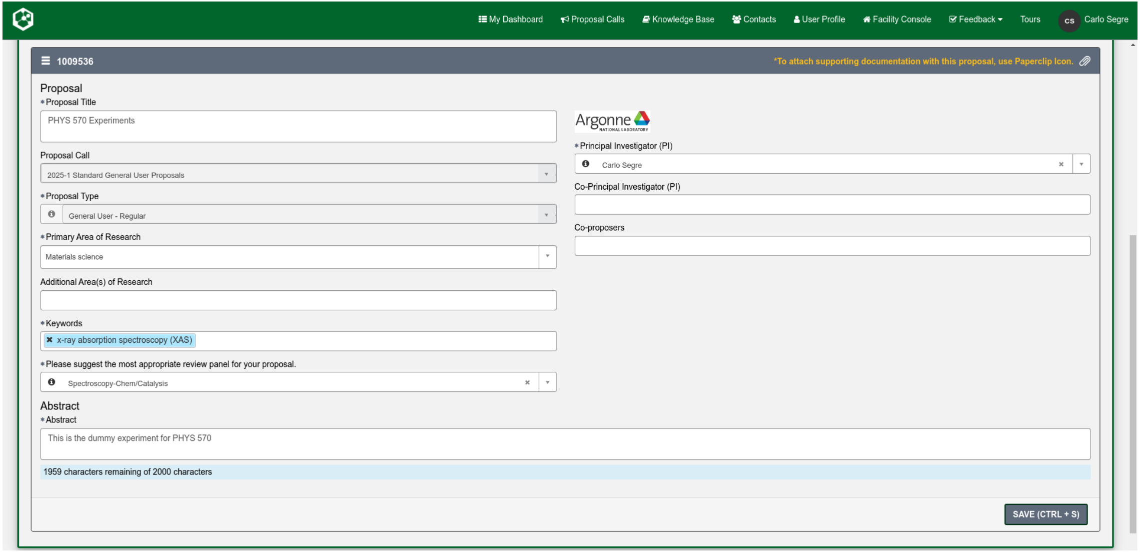
Feature Beamlines | Contact Info | Beamlines

Title	Types	Proposal Cycles	Deadline	Proposal Call Status
2025-1 eBERlight Macromolecular Crystallography	General User - Macromolecular Crystallography	APS: 2025-1	10/25/2024 21:59:59	SUBMIT A PROPOSAL
2025-1 Partner User Proposals (PUP)	Partner Proposals	APS: 2025-1	10/25/2024 21:59:59	SUBMIT A PROPOSAL
2025-1 eBERlight General User	General User - Regular	APS: 2025-1	10/25/2024 21:59:59	SUBMIT A PROPOSAL
2025-1 Standard General User Proposals	General User - Regular	APS: 2025-1	10/25/2024 21:59:59	SUBMIT A PROPOSAL
2025-1 Macromolecular Crystallography Proposals	General User - Macromolecular Crystallography	APS: 2025-1	10/25/2024 21:59:59	SUBMIT A PROPOSAL
2024-3 Standard General User - Rapid Access Proposals	General User - Rapid Access	APS: 2024-3	12/18/2024 21:59:59	SUBMIT A PROPOSAL
2024-3 CAT Member Proposals	CAT Member	APS: 2025-1, APS: 2024-3	12/18/2024 21:59:59	SUBMIT A PROPOSAL
2024-3 Resource Staff Proposals (Includes CAT and APS Staff)	Resource Staff	APS: 2025-1, APS: 2024-3	12/18/2024 21:59:59	SUBMIT A PROPOSAL
2024-3 Macromolecular Crystallography Proposals	General User - Macromolecular Crystallography	APS: 2025-1, APS: 2024-3	12/18/2024 21:59:59	SUBMIT A PROPOSAL
2025-1 Resource Staff Proposals (Includes CAT and APS Staff)	Resource Staff	APS: 2025-1	04/17/2025 21:59:59	SUBMIT A PROPOSAL
2025-1 CAT Member Proposals	CAT Member	APS: 2025-1	04/17/2025 21:59:59	SUBMIT A PROPOSAL

Enter basic information and an abstract



Enter basic information and an abstract



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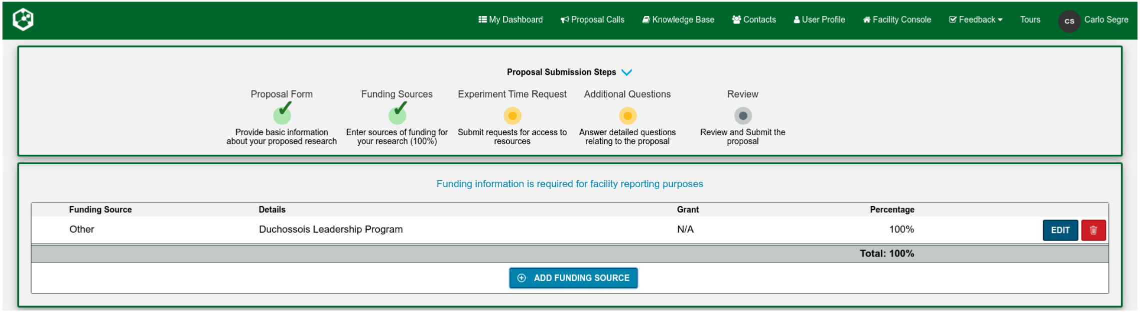

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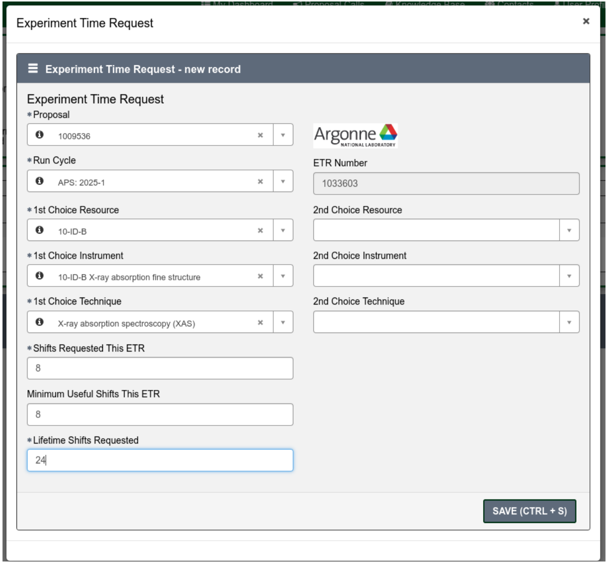

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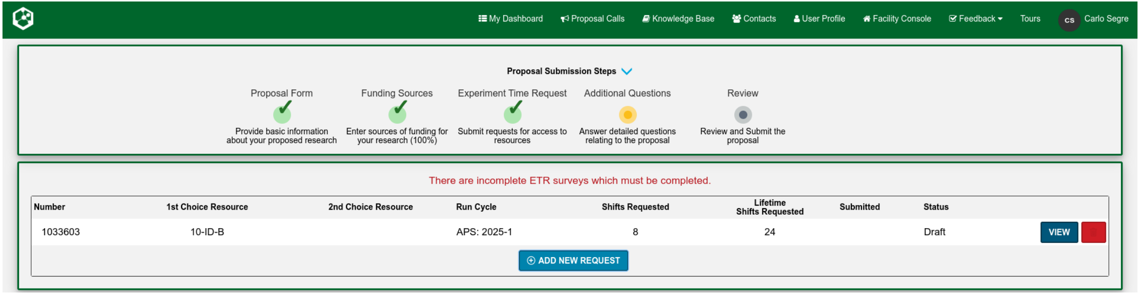

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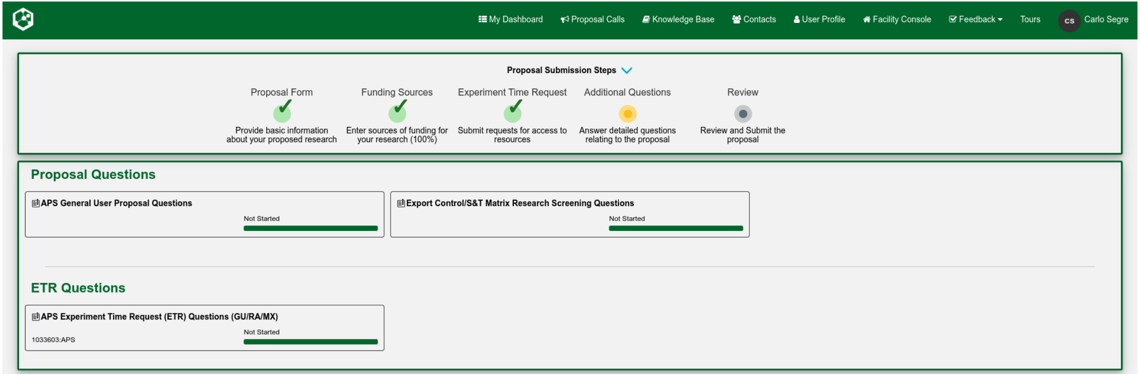


Register & log into the APS Portal

1009536 APS General User Proposal Questions

General

*What mode(s) of access would you consider for this work? (Note: not all beamlines support all modes of access, choose all that apply.)

- ☐ Remote
- ☐ Mail-in
- ☐ On-site

*Will the data collected be considered proprietary (e.g., work that will not be made available in the open literature)?

- ○ yes
- ○ no

*Have you spoken to a beamline staff member?

- ○ yes
- ○ no

*Is this research required for a student's thesis?

- ○ yes
- ○ no

*Is this proposal related to another proposal?

- ○ yes
- ○ no

Register & log into the APS Portal

* Did you previously receive experiment time at APS for this research?

- yes
- no

* Will you be requesting beam time at APS sector 35, the Dynamic Compression Sector (DCS)?

- yes
- no

Technical

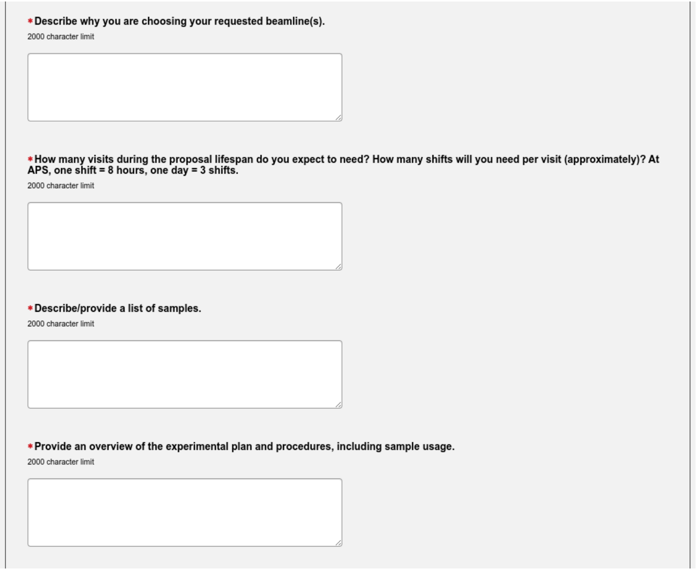
* What is the scientific or technical purpose and importance of the proposed research?

2000 character limit

* Why do you need the APS for this research?

2000 character limit

Register & log into the APS Portal



*Describe why you are choosing your requested beamline(s).

2000 character limit

*How many visits during the proposal lifespan do you expect to need? How many shifts will you need per visit (approximately)? At APS, one shift = 8 hours, one day = 3 shifts.

2000 character limit

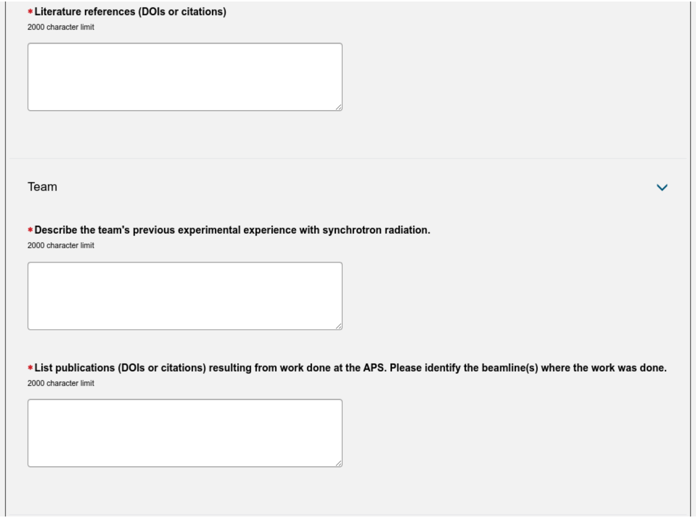
*Describe/provide a list of samples.

2000 character limit

*Provide an overview of the experimental plan and procedures, including sample usage.

2000 character limit

Register & log into the APS Portal



* Literature references (DOIs or citations)

2000 character limit

Team

* Describe the team's previous experimental experience with synchrotron radiation.

2000 character limit

* List publications (DOIs or citations) resulting from work done at the APS. Please identify the beamline(s) where the work was done.

2000 character limit

Register & log into the APS Portal

Safety

*Does this research involve the use of radioactive samples/materials, sealed sources, or x-ray generating devices?**

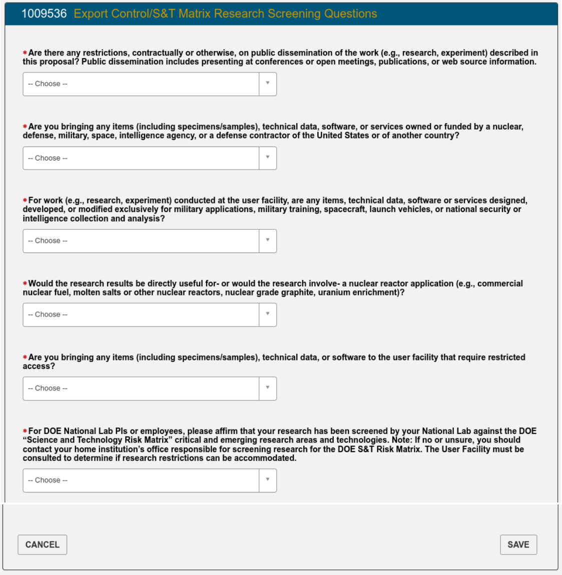
- ○ yes
- ○ no

*Does this research involve the use of any of the following (pick all that apply):**

- ☐ explosives or energetic materials
- ☐ a new class 3 or class 4 laser that has not been approved by the Argonne Laser Safety Officer
- ☐ nanoparticles (one or more dimensions is 100 nm or less), including thin films, powder, and solutions
- ☐ samples/materials that require a BSL-2 (biosafety level) facility
- ☐ human subjects or human tissues, body fluids, or cells in culture
- ☐ plant pathogens, soil microbes, animals, insects, or insect/animal tissues, body fluids, matter, cells in culture
- ☐ none

CANCEL SAVE

Register & log into the APS Portal



1009536 Export Control/S&T Matrix Research Screening Questions

*Are there any restrictions, contractually or otherwise, on public dissemination of the work (e.g., research, experiment) described in this proposal? Public dissemination includes presenting at conferences or open meetings, publications, or web source information.

-- Choose --

*Are you bringing any items (including specimens/samples), technical data, software, or services owned or funded by a nuclear, defense, military, space, intelligence agency, or a defense contractor of the United States or of another country?

-- Choose --

*For work (e.g., research, experiment) conducted at the user facility, are any items, technical data, software or services designed, developed, or modified exclusively for military applications, military training, spacecraft, launch vehicles, or national security or intelligence collection and analysis?

-- Choose --

*Would the research results be directly useful for- or would the research involve- a nuclear reactor application (e.g., commercial nuclear fuel, molten salts or other nuclear reactors, nuclear grade graphite, uranium enrichment)?

-- Choose --

*Are you bringing any items (including specimens/samples), technical data, or software to the user facility that require restricted access?

-- Choose --

*For DOE National Lab PIs or employees, please affirm that your research has been screened by your National Lab against the DOE "Science and Technology Risk Matrix" critical and emerging research areas and technologies. Note: If no or unsure, you should contact your home institution's office responsible for screening research for the DOE S&T Risk Matrix. The User Facility must be consulted to determine if research restrictions can be accommodated.

-- Choose --

CANCEL SAVE

Register & log into the APS Portal

1033603:APS Experiment Time Request (ETR) Details

If you will require use of laboratory space during the requested scheduling period, provide details here.

2000 character limit

***Will you be bringing any electrical or hazardous equipment to the facility during this scheduling period?**

- ○ Yes
- ○ No

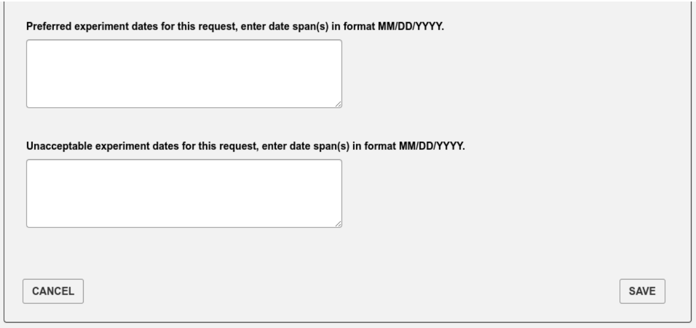
***Would the work associated with this request for time involve any of the following (choose all that apply):**

- ☐ Pure gases (or single gas component) greater than 1.3 ft3, 36 L or 1 lb. (inert gasses excluded)
- ☐ Liquids greater than 5 gal. or 19 L
- ☐ Solids greater than 18.1 kg or 40 lbs.
- ☐ None of these apply

Answer this question only for second or later ETRs: List any new publications resulting from work at APS (DOIs or citations). Please identify the beamline(s) where the work was done.

2000 character limit

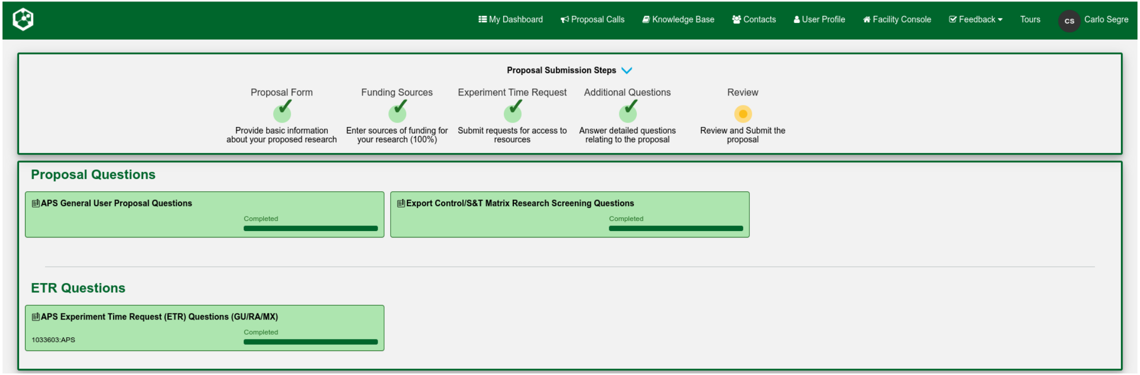
Register & log into the APS Portal


Preferred experiment dates for this request, enter date span(s) in format MM/DD/YYYY.

Unacceptable experiment dates for this request, enter date span(s) in format MM/DD/YYYY.

CANCEL SAVE

Register & log into the APS Portal



Answer the 7 important questions (2000 characters each)

What is the scientific or technical purpose and importance of the proposed research?

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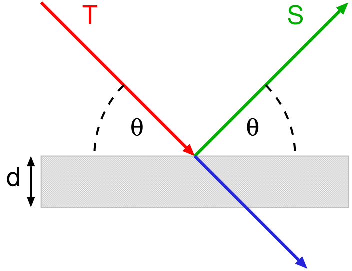
Describe the team's previous experimental experience with synchrotron radiation.

Darwin approach review

$$g = \frac{1}{m}\frac{2d^2 r_0}{v_c}|F| = \frac{\lambda r_0 d}{v_c \sin\theta}|F|$$



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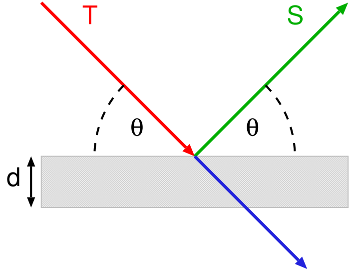
$$g = \frac{1}{m}\frac{2d^2 r_0}{v_c}|F| = \frac{\lambda r_0 d}{v_c \sin\theta}|F|$$

since $v_c \sim d^3$ then $g \sim r_0/d \approx 10^{-5}$



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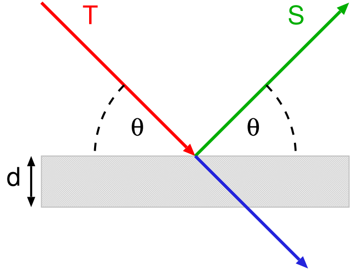

the transmitted wave is equal in amplitude to the incident wave but gains a phase shift as it passes through the layer

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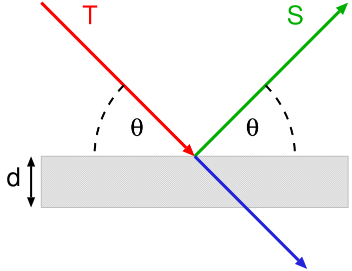

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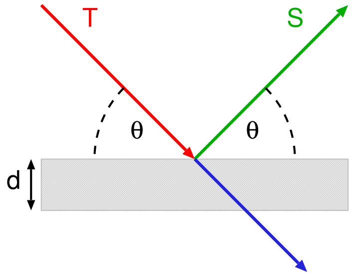

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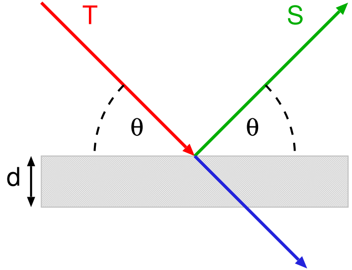
from Chapter 3

$$g_0 = \frac{\lambda \rho_{at} f^0(0) r_0 d}{\sin\theta}$$



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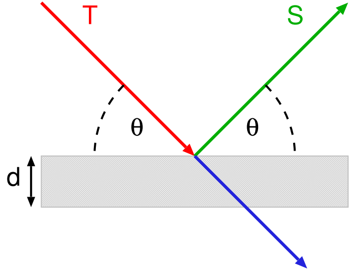
where $|F_0| = \rho_{at} f^0(0) v_c$ is the unit cell structure factor in the forward direction at $Q = \theta = 0$



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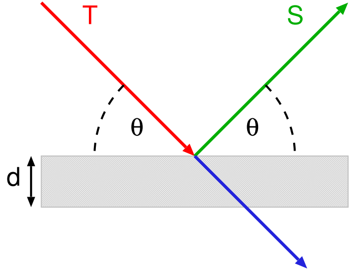
this can be rewritten in terms of g as



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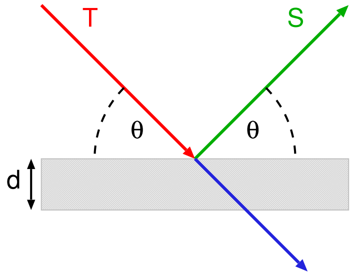
$$g_0 = g\frac{|F_0|}{|F|}$$



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Kinematical reflection

Now extend this model to N layers to get the kinematical scattering approximation as long as the total scattering is weak, $Ng \ll 1$.

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Proceed by adding reflectivity from each layer with the usual phase factor

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where the x-rays pass through each layer twice

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these N unit cell layers will give a reciprocal lattice with points at multiples of $G = 2\pi/d$

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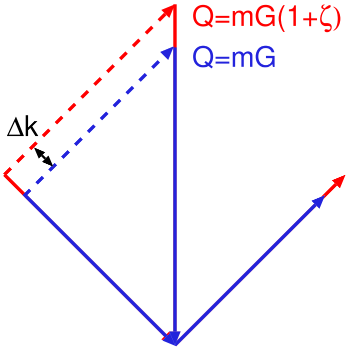


$$r_N(Q) = -ig\sum_{j=0}^{N-1} e^{iQdj}e^{-ig_0 j}e^{-ig_0 j} = -ig\sum_{j=0}^{N-1} e^{i(Qd-2g_0)j}$$

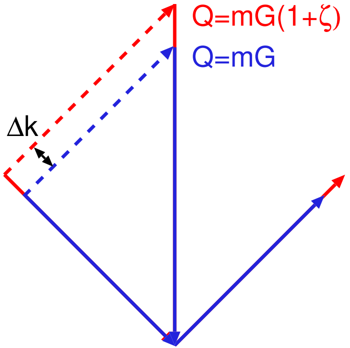
where the x-rays pass through each layer twice

these N unit cell layers will give a reciprocal lattice with points at multiples of $G = 2\pi/d$ we are interested in small deviations from the Bragg condition:

$$\zeta = \frac{\Delta Q}{Q} = \frac{\Delta k}{k} = \frac{\Delta \mathcal{E}}{\mathcal{E}} = \frac{\Delta\lambda}{\lambda}$$

Multiple layer reflection

$$r_N(Q) = -ig \sum_{j=0}^{N-1} e^{i(Qd-2g_0)j}$$



Multiple layer reflection

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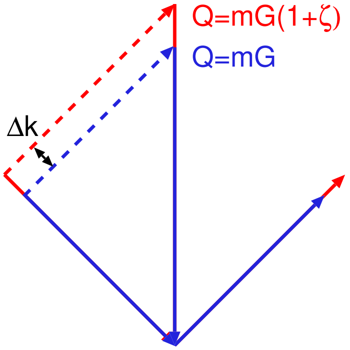

The term in the phase factor now becomes

$$Qd - 2g_0$$

Multiple layer reflection

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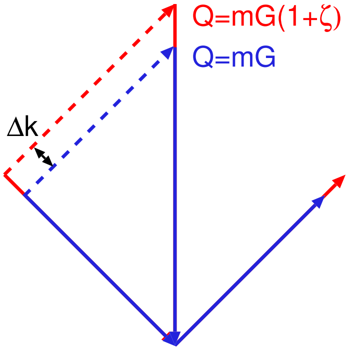

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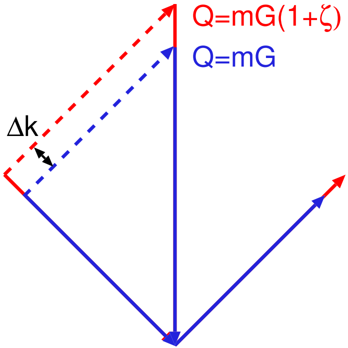

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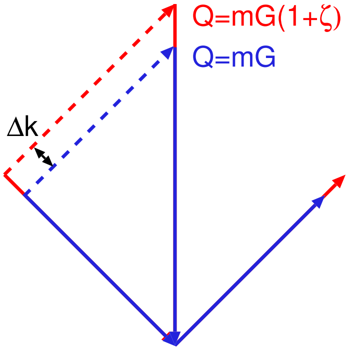

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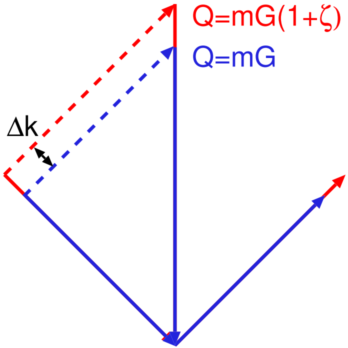

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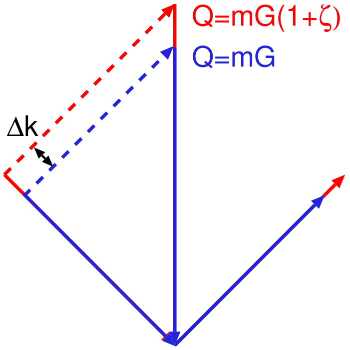

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As the crystal becomes infinite ($N \to \infty$) this kinematical approximation breaks down because $gN \sim 1$

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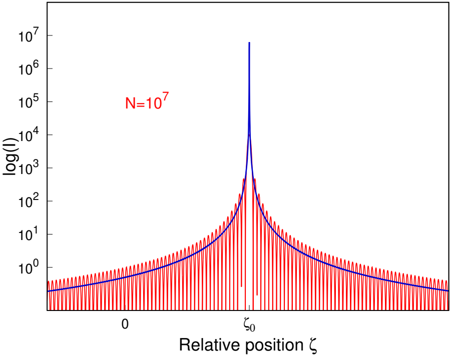

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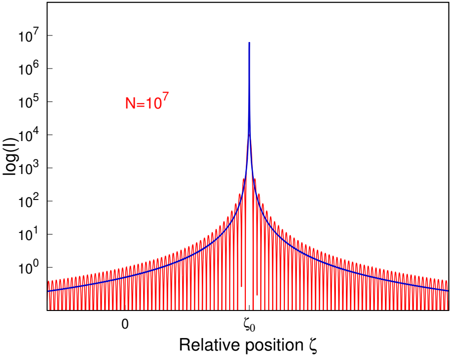

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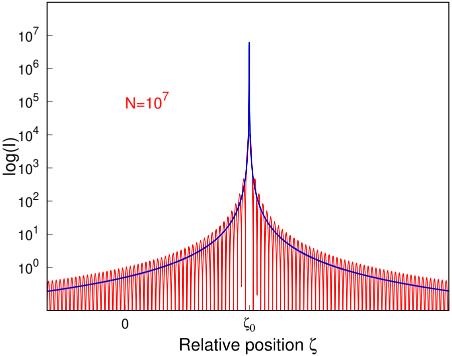

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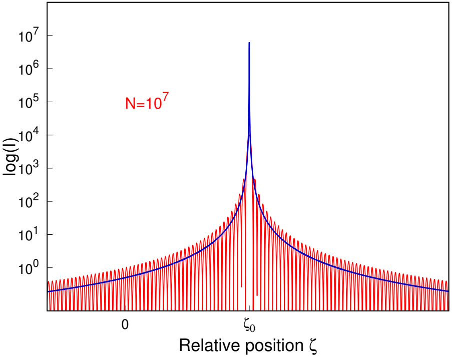

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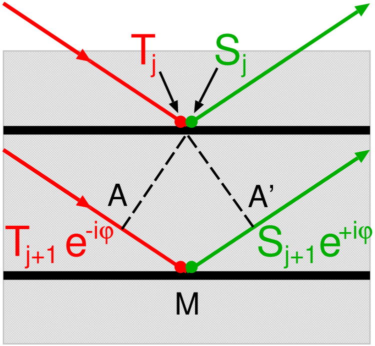
The kinematical limit clearly breaks down near ζ_0 so we need a dynamical diffraction theory

Reflectivity of a perfect crystal

In a perfect crystal, there are always two wavefields, the T wave which propagates in the direction of the incident beam and the S wave in the direction of the reflected wave



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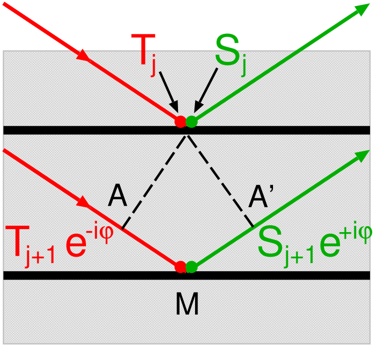

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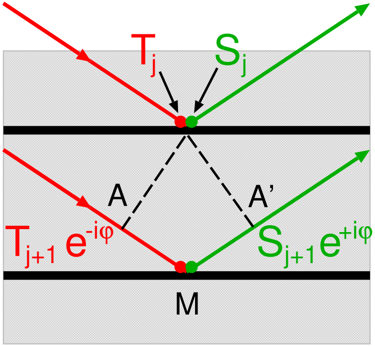

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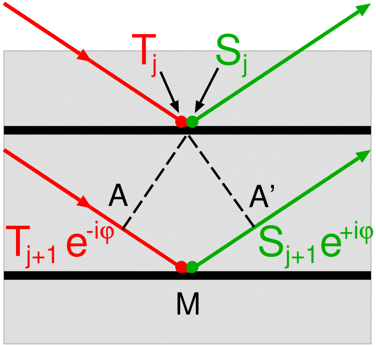

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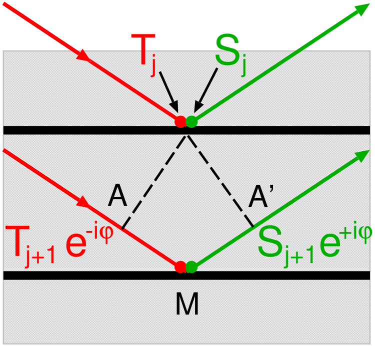

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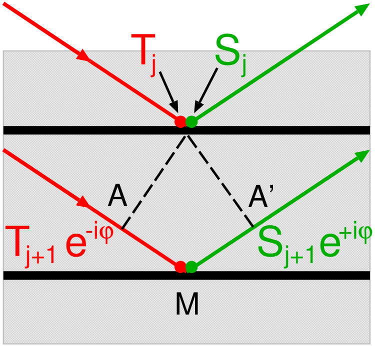

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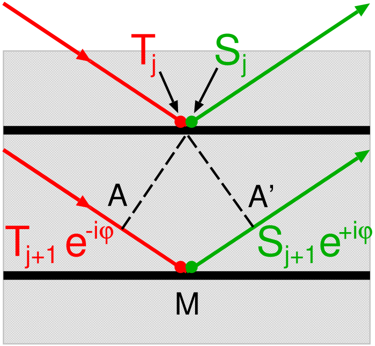

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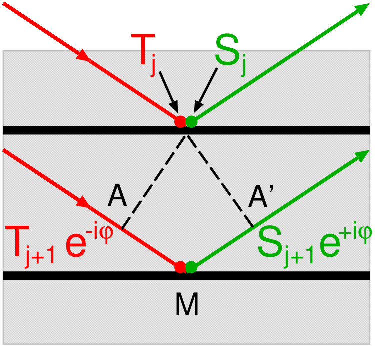

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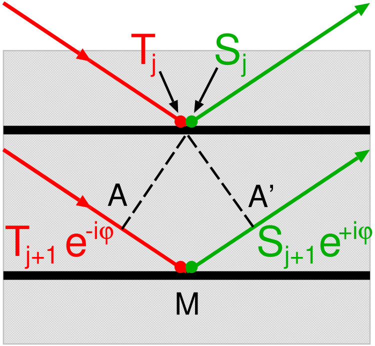
Difference equation

Let T_j and S_j be the fields just above layer j.



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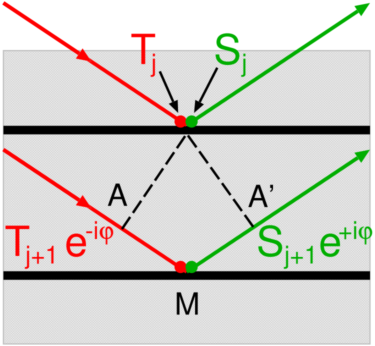

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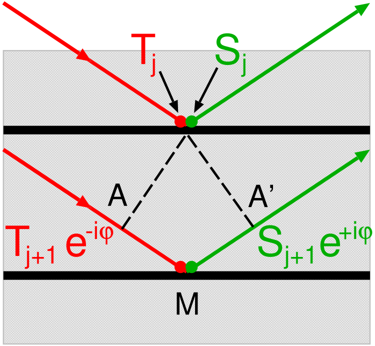

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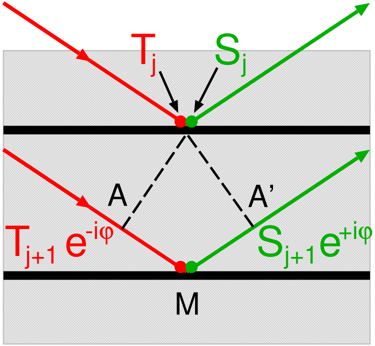

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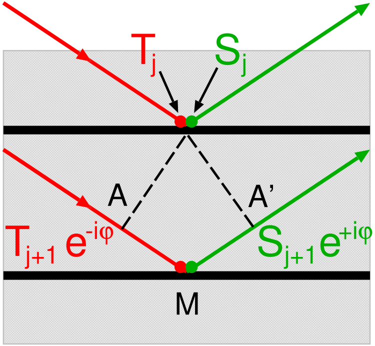

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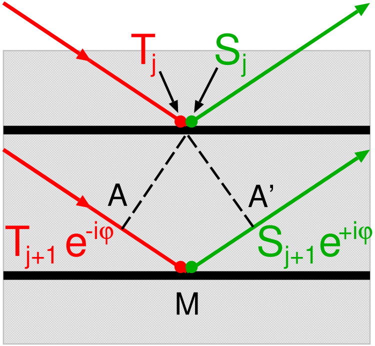

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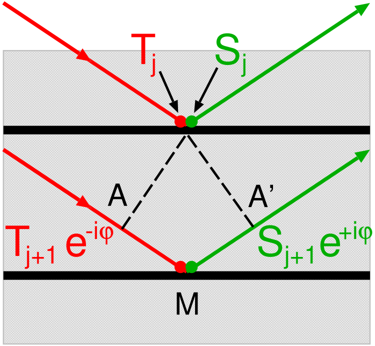
at point M, just above the $j+1^{th}$ layer, we have the scattered field S_{j+1} and at point A' it is $S_{j+1}e^{i\phi}$

but this must be equal to the field S_j just after passing up through the j^{th} layer which applies a phase shift plus the small part of the T_j field reflected from the top of the j^{th} layer



similarly we can write an equation for T_{j+1} just below the j^{th} plane

$$S_j = -ig\,T_{j+1} + (1 - ig_0)S_{j+1}e^{i\phi}$$

$$(1 - ig_0)T_j = T_{j+1}e^{-i\phi} + ig\,S_{j+1}e^{i\phi}$$

these coupled equations must be solved for an infinite stack of atomic layers

Separation of T & S fields

$$S_j = -ig T_{j+1} + (1 - ig_0) S_{j+1} e^{i\phi}, \qquad (1 - ig_0) T_j = T_{j+1} e^{-i\phi} + ig S_{j+1} e^{i\phi}$$

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Rearranging the equation for T_j (top right)

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$$ig\,S_{j+1} = (1 - ig_0)\,T_j e^{-i\phi} - T_{j+1}e^{-i2\phi}$$

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$$S_j = -ig\,T_{j+1} + (1 - ig_0)S_{j+1}e^{i\phi}, \qquad (1 - ig_0)\,T_j = T_{j+1}e^{-i\phi} + ig\,S_{j+1}e^{i\phi}$$

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shifting up by one plane: $j + 1 \rightarrow j$ and $j \rightarrow j - 1$

Separation of T & S fields

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$$ig\,S_{j+1} = (1-ig_0)\,T_j e^{-i\phi} - T_{j+1}e^{-i2\phi}$$

shifting up by one plane: $j+1 \to j$ and $j \to j-1$

$$ig\,S_j = (1-ig_0)\,T_{j-1}e^{-i\phi} - T_j e^{-i2\phi}$$

Separation of T & S fields

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Separation of T & S fields

$$S_j = -ig\,T_{j+1} + (1 - ig_0)S_{j+1}e^{i\phi}, \qquad (1 - ig_0)\,T_j = T_{j+1}e^{-i\phi} + ig\,S_{j+1}e^{i\phi}$$

Rearranging the equation for T_j (top right)

$$ig\,S_{j+1} = (1 - ig_0)\,T_j e^{-i\phi} - T_{j+1}e^{-i2\phi}$$

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$$(1 - ig_0)e^{-i\phi}\left[T_{j+1} + T_{j-1}\right] = \left[g^2 + (1 - ig_0)^2 + e^{-i2\phi}\right]T_j$$

Separation of T & S fields

$$S_j = -ig\,T_{j+1} + (1 - ig_0)S_{j+1}e^{i\phi}, \qquad (1 - ig_0)\,T_j = T_{j+1}e^{-i\phi} + ig\,S_{j+1}e^{i\phi}$$

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the fields T_j and T_{j+1} are out of phase by nearly $m\pi$ (top right equation) since g and g_0 are very small and the T wave field must attenuate as it penetrates deeper into the crystal so our trial solution is

Separation of T & S fields

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$$ig\,S_{j+1} = (1-ig_0)\,T_j e^{-i\phi} - T_{j+1}e^{-i2\phi}$$

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$$(1-ig_0)\,T_{j-1}e^{-i\phi} - T_j e^{-i2\phi} = g^2\,T_j + (1-ig_0)\left[(1-ig_0)\,T_j - T_{j+1}e^{-i\phi}\right]$$

$$(1-ig_0)e^{-i\phi}\left[T_{j+1} + T_{j-1}\right] = \left[g^2 + (1-ig_0)^2 + e^{-i2\phi}\right] T_j$$

the fields T_j and T_{j+1} are out of phase by nearly $m\pi$ (top right equation) since g and g_0 are very small and the T wave field must attenuate as it penetrates deeper into the crystal so our trial solution is

$$T_{j+1} = e^{-\eta}e^{im\pi}\,T_j$$

Solving for the T field

$$(1 - ig_0)e^{-i\phi}\left[T_{j+1} + T_{j-1}\right] = \left[g^2 + (1 - ig_0)^2 + e^{-i2\phi}\right] T_j$$

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Solving for the T field

$$(1 - ig_0)e^{-i\phi}\left[T_{j+1} + T_{j-1}\right] = \left[g^2 + (1 - ig_0)^2 + e^{-i2\phi}\right] T_j$$

With the trial solution ,

Solving for the T field

$$(1 - ig_0)e^{-i\phi}\left[T_{j+1} + T_{j-1}\right] = \left[g^2 + (1 - ig_0)^2 + e^{-i2\phi}\right] T_j$$

With the trial solution $$T_{j+1} = e^{-\eta} e^{im\pi} T_j,$$

Solving for the T field

$$(1 - ig_0)e^{-i\phi}\left[T_{j+1} + T_{j-1}\right] = \left[g^2 + (1 - ig_0)^2 + e^{-i2\phi}\right] T_j$$

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Solving for the T field

$$(1 - ig_0)e^{-i\phi}\left[T_{j+1} + T_{j-1}\right] = \left[g^2 + (1 - ig_0)^2 + e^{-i2\phi}\right] T_j$$

With the trial solution $\quad T_{j+1} = e^{-\eta}e^{im\pi}\,T_j, \quad T_{j-1} = e^{\eta}e^{-im\pi}\,T_j$

and substituting this solution into the defining equation for T

Solving for the T field

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$$(1 - ig_0)e^{-i\phi}\left[e^{-\eta} e^{im\pi} T_j + e^{\eta} e^{-im\pi} T_j\right] = \left[g^2 + (1 - ig_0)^2 + e^{-i2\phi}\right] T_j$$

Solving for the T field

$$(1 - ig_0)e^{-i\phi}\left[T_{j+1} + T_{j-1}\right] = \left[g^2 + (1 - ig_0)^2 + e^{-i2\phi}\right] T_j$$

With the trial solution $T_{j+1} = e^{-\eta}e^{im\pi}\,T_j, \qquad T_{j-1} = e^{\eta}e^{-im\pi}\,T_j$

and substituting this solution into the defining equation for T and noting that $\phi \equiv m\pi + \Delta$

$$(1 - ig_0)e^{-i\phi}\left[e^{-\eta}e^{im\pi}\,T_j + e^{\eta}e^{-im\pi}\,T_j\right] = \left[g^2 + (1 - ig_0)^2 + e^{-i2\phi}\right] T_j$$

Solving for the T field

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$$(1 - ig_0)e^{-im\pi}e^{-i\Delta}\left[e^{-\eta}e^{im\pi} + e^{\eta}e^{-im\pi}\right] = g^2 + (1 - ig_0)^2 + e^{-i2m\pi}e^{-i2\Delta}$$

Solving for the T field

$$(1-ig_0)e^{-i\phi}\left[T_{j+1}+T_{j-1}\right]=\left[g^2+(1-ig_0)^2+e^{-i2\phi}\right]T_j$$

With the trial solution $T_{j+1}=e^{-\eta}e^{im\pi}T_j, \quad T_{j-1}=e^{\eta}e^{-im\pi}T_j$

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$$(1-ig_0)e^{-i\phi}\left[e^{-\eta}e^{im\pi}T_j+e^{\eta}e^{-im\pi}T_j\right]=\left[g^2+(1-ig_0)^2+e^{-i2\phi}\right]T_j$$

$$(1-ig_0)\cancel{e^{-im\pi}}e^{-i\Delta}\left[e^{-\eta}\cancel{e^{im\pi}}+e^{\eta}\cancel{e^{-im\pi}}\right]=g^2+(1-ig_0)^2+\cancel{e^{-i2m\pi}}e^{-i2\Delta}$$

$$(1-ig_0)e^{-i\Delta}\left[e^{-\eta}+e^{\eta}\right]=g^2+(1-ig_0)^2+e^{-i2\Delta}$$

Solving for the T field

$$(1-ig_0)e^{-i\phi}\left[T_{j+1}+T_{j-1}\right]=\left[g^2+(1-ig_0)^2+e^{-i2\phi}\right]T_j$$

With the trial solution $T_{j+1}=e^{-\eta}e^{im\pi}T_j, \quad T_{j-1}=e^{\eta}e^{-im\pi}T_j$

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$$(1-ig_0)e^{-i\phi}\left[e^{-\eta}e^{im\pi}T_j+e^{\eta}e^{-im\pi}T_j\right]=\left[g^2+(1-ig_0)^2+e^{-i2\phi}\right]T_j$$

$$(1-ig_0)\cancel{e^{-im\pi}}e^{-i\Delta}\left[e^{-\eta}\cancel{e^{im\pi}}+e^{\eta}\cancel{e^{-im\pi}}\right]=g^2+(1-ig_0)^2+\cancel{e^{-i2m\pi}}e^{-i2\Delta}$$

$$(1-ig_0)e^{-i\Delta}\left[e^{-\eta}+e^{\eta}\right]=g^2+(1-ig_0)^2+e^{-i2\Delta}$$

assuming that g, g_0, and Δ are very small quantities, we can expand

Solving for the T field

$$(1 - ig_0)e^{-i\phi}\left[T_{j+1} + T_{j-1}\right] = \left[g^2 + (1 - ig_0)^2 + e^{-i2\phi}\right] T_j$$

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$$(1 - ig_0)\cancel{e^{-im\pi}}e^{-i\Delta}\left[e^{-\eta}\cancel{e^{im\pi}} + e^{\eta}\cancel{e^{-im\pi}}\right] = g^2 + (1 - ig_0)^2 + \cancel{e^{-i2m\pi}}e^{-i2\Delta}$$

$$(1 - ig_0)e^{-i\Delta}\left[e^{-\eta} + e^{\eta}\right] = g^2 + (1 - ig_0)^2 + e^{-i2\Delta}$$

assuming that g, g_0, and Δ are very small quantities, we can expand

$$(1 - ig_0)(1 - i\Delta - \frac{\Delta^2}{2})\left[(1 - \eta + \frac{\eta^2}{2}) + (1 + \eta + \frac{\eta^2}{2})\right]$$

$$\approx g^2 + (1 - 2ig_0 - g_0^2) + (1 - i2\Delta - 2\Delta^2)$$

Solving for the T field

$$(1 - ig_0)(1 - i\Delta - \frac{\Delta^2}{2})\left[(1 - \eta + \frac{\eta^2}{2}) + (1 + \eta + \frac{\eta^2}{2})\right]$$
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Solving for the T field

$$(1 - ig_0)(1 - i\Delta - \frac{\Delta^2}{2})\left[(1 - \eta + \frac{\eta^2}{2}) + (1 + \eta + \frac{\eta^2}{2})\right]$$
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Cancelling and expanding all products keeping only second order terms

Solving for the T field

$$(1 - ig_0)(1 - i\Delta - \frac{\Delta^2}{2})\left[(1 - \eta + \frac{\eta^2}{2}) + (1 + \eta + \frac{\eta^2}{2})\right]$$
$$\approx g^2 + (1 - 2ig_0 - g_0^2) + (1 - i2\Delta - 2\Delta^2)$$

Cancelling and expanding all products keeping only second order terms

$$(1 - ig_0 - i\Delta - g_0\Delta - \frac{\Delta^2}{2})(2 + \eta^2) \approx g^2 + 2 - 2ig_0 - 2i\Delta - g_0^2 - 2\Delta^2$$

Solving for the T field

$$(1 - ig_0)(1 - i\Delta - \frac{\Delta^2}{2})\left[(1 - \eta + \frac{\eta^2}{2}) + (1 + \eta + \frac{\eta^2}{2})\right]$$
$$\approx g^2 + (1 - 2ig_0 - g_0^2) + (1 - i2\Delta - 2\Delta^2)$$

Cancelling and expanding all products keeping only second order terms

$$(1 - ig_0 - i\Delta - g_0\Delta - \frac{\Delta^2}{2})(2 + \eta^2) \approx g^2 + 2 - 2ig_0 - 2i\Delta - g_0^2 - 2\Delta^2$$
$$2 - 2ig_0 - 2i\Delta - 2g_0\Delta - \Delta^2 + \eta^2 \approx g^2 + 2 - 2ig_0 - 2i\Delta - g_0^2 - 2\Delta^2$$

Solving for the T field

$$(1 - ig_0)(1 - i\Delta - \frac{\Delta^2}{2})\left[(1 - \eta + \frac{\eta^2}{2}) + (1 + \eta + \frac{\eta^2}{2})\right]$$
$$\approx g^2 + (1 - 2ig_0 - g_0^2) + (1 - i2\Delta - 2\Delta^2)$$

Cancelling and expanding all products keeping only second order terms

$$(1 - ig_0 - i\Delta - g_0\Delta - \frac{\Delta^2}{2})(2 + \eta^2) \approx g^2 + 2 - 2ig_0 - 2i\Delta - g_0^2 - 2\Delta^2$$
$$\cancel{2} - \cancel{2ig_0} - \cancel{2i\Delta} - 2g_0\Delta - \Delta^2 + \eta^2 \approx g^2 + \cancel{2} - \cancel{2ig_0} - \cancel{2i\Delta} - g_0^2 - 2\Delta^2$$

Solving for the T field

$$(1 - ig_0)(1 - i\Delta - \frac{\Delta^2}{2})\left[(1 - \eta + \frac{\eta^2}{2}) + (1 + \eta + \frac{\eta^2}{2})\right] \approx g^2 + (1 - 2ig_0 - g_0^2) + (1 - i2\Delta - 2\Delta^2)$$

Cancelling and expanding all products keeping only second order terms

$$(1 - ig_0 - i\Delta - g_0\Delta - \frac{\Delta^2}{2})(2 + \eta^2) \approx g^2 + 2 - 2ig_0 - 2i\Delta - g_0^2 - 2\Delta^2$$

$$\cancel{2} - \cancel{2ig_0} - \cancel{2i\Delta} - 2g_0\Delta - \Delta^2 + \eta^2 \approx g^2 + \cancel{2} - \cancel{2ig_0} - \cancel{2i\Delta} - g_0^2 - 2\Delta^2$$

$$\eta^2 \approx g^2 - g_0^2 + 2g_0\Delta - \Delta^2$$

Solving for the T field

$$(1 - ig_0)(1 - i\Delta - \frac{\Delta^2}{2})\left[(1 - \eta + \frac{\eta^2}{2}) + (1 + \eta + \frac{\eta^2}{2})\right]$$
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Cancelling and expanding all products keeping only second order terms

$$(1 - ig_0 - i\Delta - g_0\Delta - \frac{\Delta^2}{2})(2 + \eta^2) \approx g^2 + 2 - 2ig_0 - 2i\Delta - g_0^2 - 2\Delta^2$$
$$\cancel{2} - \cancel{2ig_0} - \cancel{2i\Delta} - 2g_0\Delta - \Delta^2 + \eta^2 \approx g^2 + \cancel{2} - \cancel{2ig_0} - \cancel{2i\Delta} - g_0^2 - 2\Delta^2$$
$$\eta^2 \approx g^2 - g_0^2 + 2g_0\Delta - \Delta^2 = g^2 - (\Delta - g_0)^2$$

Solving for the T field

$$(1 - ig_0)(1 - i\Delta - \frac{\Delta^2}{2})\left[(1 - \eta + \frac{\eta^2}{2}) + (1 + \eta + \frac{\eta^2}{2})\right]$$
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Cancelling and expanding all products keeping only second order terms

$$(1 - ig_0 - i\Delta - g_0\Delta - \frac{\Delta^2}{2})(2 + \eta^2) \approx g^2 + 2 - 2ig_0 - 2i\Delta - g_0^2 - 2\Delta^2$$
$$\cancel{2} - \cancel{2ig_0} - \cancel{2i\Delta} - 2g_0\Delta - \Delta^2 + \eta^2 \approx g^2 + \cancel{2} - \cancel{2ig_0} - \cancel{2i\Delta} - g_0^2 - 2\Delta^2$$
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The solution for the attenuation factor of the transmitted field is thus

Solving for the T field

$$(1 - ig_0)(1 - i\Delta - \frac{\Delta^2}{2})\left[(1 - \eta + \frac{\eta^2}{2}) + (1 + \eta + \frac{\eta^2}{2})\right]$$
$$\approx g^2 + (1 - 2ig_0 - g_0^2) + (1 - i2\Delta - 2\Delta^2)$$

Cancelling and expanding all products keeping only second order terms

$$(1 - ig_0 - i\Delta - g_0\Delta - \frac{\Delta^2}{2})(2 + \eta^2) \approx g^2 + 2 - 2ig_0 - 2i\Delta - g_0^2 - 2\Delta^2$$
$$\cancel{2} - \cancel{2ig_0} - \cancel{2i\Delta} - 2g_0\Delta - \Delta^2 + \eta^2 \approx g^2 + \cancel{2} - \cancel{2ig_0} - \cancel{2i\Delta} - g_0^2 - 2\Delta^2$$
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The solution for the attenuation factor of the transmitted field is thus

$$i\eta = \pm\sqrt{(\Delta - g_0) - g^2}$$

Solving for the T field

$$(1 - ig_0)(1 - i\Delta - \frac{\Delta^2}{2}) \left[(1 - \eta + \frac{\eta^2}{2}) + (1 + \eta + \frac{\eta^2}{2}) \right]$$
$$\approx g^2 + (1 - 2ig_0 - g_0^2) + (1 - i2\Delta - 2\Delta^2)$$

Cancelling and expanding all products keeping only second order terms

$$(1 - ig_0 - i\Delta - g_0\Delta - \frac{\Delta^2}{2})(2 + \eta^2) \approx g^2 + 2 - 2ig_0 - 2i\Delta - g_0^2 - 2\Delta^2$$
$$\cancel{2} - \cancel{2ig_0} - \cancel{2i\Delta} - 2g_0\Delta - \Delta^2 + \eta^2 \approx g^2 + \cancel{2} - \cancel{2ig_0} - \cancel{2i\Delta} - g_0^2 - 2\Delta^2$$
$$\eta^2 \approx g^2 - g_0^2 + 2g_0\Delta - \Delta^2 = g^2 - (\Delta - g_0)^2$$

The solution for the attenuation factor of the transmitted field is thus

$$i\eta = \pm\sqrt{(\Delta - g_0) - g^2}$$

with fields

Solving for the T field

$$(1-ig_0)(1-i\Delta-\frac{\Delta^2}{2})\left[(1-\eta+\frac{\eta^2}{2})+(1+\eta+\frac{\eta^2}{2})\right]$$
$$\approx g^2+(1-2ig_0-g_0^2)+(1-i2\Delta-2\Delta^2)$$

Cancelling and expanding all products keeping only second order terms

$$(1-ig_0-i\Delta-g_0\Delta-\frac{\Delta^2}{2})(2+\eta^2)\approx g^2+2-2ig_0-2i\Delta-g_0^2-2\Delta^2$$
$$\cancel{2}-\cancel{2ig_0}-\cancel{2i\Delta}-2g_0\Delta-\Delta^2+\eta^2\approx g^2+\cancel{2}-\cancel{2ig_0}-\cancel{2i\Delta}-g_0^2-2\Delta^2$$
$$\eta^2\approx g^2-g_0^2+2g_0\Delta-\Delta^2=g^2-(\Delta-g_0)^2$$

The solution for the attenuation factor of the transmitted field is thus

$$i\eta=\pm\sqrt{(\Delta-g_0)-g^2}$$

with fields

$$T_{j+1}=e^{-\eta}e^{im\pi}T_j,$$

Solving for the T field

$$(1 - ig_0)(1 - i\Delta - \frac{\Delta^2}{2})\left[(1 - \eta + \frac{\eta^2}{2}) + (1 + \eta + \frac{\eta^2}{2})\right]$$
$$\approx g^2 + (1 - 2ig_0 - g_0^2) + (1 - i2\Delta - 2\Delta^2)$$

Cancelling and expanding all products keeping only second order terms

$$(1 - ig_0 - i\Delta - g_0\Delta - \frac{\Delta^2}{2})(2 + \eta^2) \approx g^2 + 2 - 2ig_0 - 2i\Delta - g_0^2 - 2\Delta^2$$
$$\cancel{2} - \cancel{2ig_0} - \cancel{2i\Delta} - 2g_0\Delta - \Delta^2 + \eta^2 \approx g^2 + \cancel{2} - \cancel{2ig_0} - \cancel{2i\Delta} - g_0^2 - 2\Delta^2$$
$$\eta^2 \approx g^2 - g_0^2 + 2g_0\Delta - \Delta^2 = g^2 - (\Delta - g_0)^2$$

The solution for the attenuation factor of the transmitted field is thus

$$i\eta = \pm\sqrt{(\Delta - g_0) - g^2}$$

with fields

$$T_{j+1} = e^{-\eta}e^{im\pi}T_j, \qquad S_{j+1} = e^{-\eta}e^{im\pi}S_j$$